

1 ÜBUNGSAUFGABEN MESK 2BKII

Ermitteln Sie die Lösungsmenge folgender LGS (jeweils als Matrix angegeben):

Bitte Probe machen.

Falls ein LGS unendlich viele Lösungen besitzt, müssen jeweils 2 Lösungen angegeben werden.

I)

1)	2)	3)	4)	5)
1 0 9 18	1 -8 0 19	4 1 0 -17	0 1 -8 17	0 7 1 11
0 1 -7 15	0 5 1 -16	-9 0 1 13	1 0 9 -16	1 -6 0 -9

6)	7)	8)	9)	10)
-9 1 0 14	1 19 -5 12	12 1 9 -18	9 8 1 -17	1 2 3 4
12 0 1 -13				2 4 6 9
				0 0 0 7

11)	12)	13)	14)
6 10 -8 4	1 -2 4 3	-4 2 16	0 3 4 13
3 5 -4 2	5 -6 3 2	6 5 8	-3 2 0 12
-18 -30 24 -12	-10 12 -6 -4	7 3 -2	-3 8 8 38

II)

Wochenrätsel bei Spiegel-Online:

<http://www.spiegel.de/wissenschaft/mensch/raetsel-der-woche-verrueckte-rechnerei-a-1111679.html>

$$8 + 11 = 310$$

$$22 + 9 = 1313$$

$$43 + 56 = 1318$$

$$72 + 19 = 5319$$

$$8 + 6 = 214$$

$$22 + 11 = ?$$

Welche Rechenmethode steckt dahinter?

Falls ja, dann wissen Sie sicher auch schon, was $22 + 11$ ergibt.

Lösungen

$$1) L = \{ (x_1; x_2; x_3) \mid x_1=18-9x_3 \wedge x_2=15+7x_3 \wedge x_3 \in \mathbb{R} \}$$

Beispiel:

$$(18;15;0) \in L$$

Probe:

$$1 \cdot 18 + 0 \cdot 15 + 9 \cdot 0 = 18 \quad \text{wahr}$$

$$0 \cdot 18 + 1 \cdot 15 - 7 \cdot 0 = 15 \quad \text{wahr}$$

$$2) L = \{ (x_1; x_2; x_3) \mid x_1=19+8x_2 \wedge x_3=-16-5x_2 \wedge x_2 \in \mathbb{R} \}$$

$$3) L = \{ (x_1; x_2; x_3) \mid x_2=-17-4x_1 \wedge x_3=13+9x_1 \wedge x_1 \in \mathbb{R} \}$$

$$4) L = \{ (x_1; x_2; x_3) \mid x_2=17+8x_3 \wedge x_1=-16-9x_3 \wedge x_3 \in \mathbb{R} \}$$

$$5) L = \{ (x_1; x_2; x_3) \mid x_3=11-7x_2 \wedge x_1=-9+6x_2 \wedge x_2 \in \mathbb{R} \}$$

$$6) L = \{ (x_1; x_2; x_3) \mid x_2=14+9x_1 \wedge x_3=-13-12x_1 \wedge x_1 \in \mathbb{R} \}$$

$$7) L = \{ (x_1; x_2; x_3) \mid x_1=12-19x_2+5x_3 \wedge x_2 \in \mathbb{R} \wedge x_3 \in \mathbb{R} \}$$

$$8) L = \{ (x_1; x_2; x_3) \mid x_2=-18-12x_1-9x_3 \wedge x_1 \in \mathbb{R} \wedge x_3 \in \mathbb{R} \}$$

$$9) L = \{ (x_1; x_2; x_3) \mid x_3=-17-9x_1-8x_2 \wedge x_1 \in \mathbb{R} \wedge x_2 \in \mathbb{R} \}$$

$$10) L = \{ \}$$

11)

x_1	x_2	x_3	b	Op	KS
6	10	-8	4	G1	12
3	5	-4	2	G2	6
-18	-30	24	-12	G3	-36
6	10	-8	4	G4=G1	12
0	0	0	0	G5=G1-2G2	0
0	0	0	0	G6=3G1+G3	0
6	10	-8	4	G7=G1	12
1	5/3	-4/3	2/3	G8=G7/6	2

$$L = \{ (x_1; x_2; x_3) \mid x_1 = 2/3 - 5/3 \cdot x_2 + 4/3 \cdot x_3 \wedge x_2 \in \mathbb{R} \wedge x_3 \in \mathbb{R} \}$$

12)

x_1	x_2	x_3	b	Op	KS
1	-2	4	3	G1	6
5	-6	3	2	G2	4
-10	12	-6	-4	G3	-8
1	-2	4	3	G4=G1	6
0	4	-17	-13	G5=-5G1+G2	-26
0	-8	34	26	G6=10G1+G3	52
17	-18	0	-1	G7=4G5+17G1	-2
0	4	-17	-13	G8=G5	-26
0	0	0	0	G8=2G5+G6	0
17	-18	0	-1	G9=G7	-2
0	4	-17	-13	G10=G8	-26
1	-18/17	0	-1/17	G11=G9/17	
0	-4/17	1	13/17	G12=G10/-17	

$$L = \{ (x_1; x_2; x_3) \mid x_1 = -1/17 + 18/17 \cdot x_2 \wedge x_3 = 13/17 + 4/17 \cdot x_2 \wedge x_2 \in \mathbb{R} \}$$

13)

x_1	x_2	b	Op	KS
-4	2	16	G1	14
6	5	8	G2	19
7	3	-2	G3	8
-4	2	16	G4=G1	14
0	16	64	G5=3G1+2G2	80
0	26	104	G6=7G1+4G3	130
32	0	-64	G7=-8G4+G5	-32
0	16	64	G8=G5	80
0	0	0	G9=13G5-8G6	0
32	0	-64	G10=G7	-2
0	16	64	G11=G8	-26
1	0	-2	G12=G10/32	-1
0	1	4	G13=G11/16	5

$$L = \{(-2; 4) \}$$

14)

x_1	x_2	x_3	b	Op	KS
0	3	4	13	G1	20
-3	2	0	12	G2	11
-3	8	8	38	G3	51
0	3	4	13	G4	20
-3	2	0	12	G5	11
0	-6	-8	-26	G6	-40
0	3	4	13	G7	20
-9	0	-8	10	G8=-2G4+3G5	-7
0	0	0	0	G9=2G4+G6	0
0	3	4	13	G10=G7	20
-9	0	-8	10	G11=G8	-7
0	1	4/3	13/3	G12=G10/3	20/3
1	0	8/9	-10/9	G13=G11/-9	-7/9

$$L = \{(x_1; x_2; x_3) \mid x_2=13/3-4/3 \cdot x_3 \wedge x_1=-10/9-8/9 \cdot x_3 \wedge x_3 \in \mathbb{R} \}$$

II)

Ansatz:

$$x + y = z$$

ist eine Abkürzung für:

$$a \cdot x^2 + bx + cy^2 + dy + e = z$$

also steht

$$8 + 11 = 310$$

$$22 + 9 = 1313$$

$$43 + 56 = 1318$$

$$72 + 19 = 5319$$

$$8 + 6 = 214$$

$$22 + 11 = ?$$

für:

$$a \cdot 8^2 + b \cdot 8 + c \cdot 11^2 + d \cdot 11 + e = 310$$

$$a \cdot 22^2 + b \cdot 22 + c \cdot 9^2 + d \cdot 9 + e = 1313$$

$$a \cdot 43^2 + b \cdot 43 + c \cdot 56^2 + d \cdot 56 + e = 1318$$

$$a \cdot 72^2 + b \cdot 72 + c \cdot 19^2 + d \cdot 19 + e = 5319$$

$$a \cdot 8^2 + b \cdot 8 + c \cdot 6^2 + d \cdot 6 + e = 214$$

also:

$$64a + 8b + 121c + 11d + 1 = 310$$

$$484a + 22b + 81c + 9d + 1 = 1313$$

$$1849a + 43b + 3136c + 56d + 1 = 1318$$

$$5184a + 72b + 361c + 19d + 1 = 5319$$

$$64a + 8b + 36c + 6d + 1 = 214$$

ergibt folgende Matrix:

$n=5$ (Zahl der Unbekannten) $m=5$ (Zahl der Gleichungen)

$$64 \ 8 \ 121 \ 11 \ 1 \ | \ 310$$

$$484 \ 22 \ 81 \ 9 \ 1 \ | \ 1313$$

$$1849 \ 43 \ 3136 \ 56 \ 1 \ | \ 1318$$

$$5184 \ 72 \ 361 \ 19 \ 1 \ | \ 5319$$

$$64 \ 8 \ 36 \ 6 \ 1 \ | \ 214$$

kann mit einer entsprechenden Software gelöst werden.

2 ÜBUNGSAUFGABEN MESK 2BK11

Ermitteln Sie die Lösungsmenge folgender LGS

<p>1)</p> $\begin{aligned} x_1 + 3x_2 - 2x_3 &= 4,5 \\ -x_1 + 2x_2 - 3x_3 &= 1,5 \\ 3x_1 - 4x_2 + 2x_3 &= 0,9 \end{aligned}$	<p>2)</p> $\begin{aligned} 1,6x_1 - 0,5x_2 + 2x_3 &= 0,1 \\ 2x_1 + 1,2x_2 - x_3 &= 1,8 \\ 0,8x_1 - 2x_2 - 5x_3 &= 7,8 \end{aligned}$	<p>3)</p> $\begin{aligned} 0,4x_1 + 0,8x_2 + 1,2x_3 &= 1,8 \\ 2,1x_1 - 1,4x_2 - 3,5x_3 &= 10,5 \\ -3x_1 - 2,5x_2 + x_3 &= -3,3 \end{aligned}$
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<p>4)</p> $\begin{aligned} x_1 - 0,5x_2 + 2x_3 &= -3 \\ 2x_1 + 1,2x_2 - x_3 &= 4 \\ 3x_1 - 2x_2 + 2,5x_3 &= -2 \end{aligned}$	<p>5)</p> $\begin{aligned} 2x_1 + 5x_2 + 2x_3 &= -4 \\ -2x_1 + 4x_2 - 5x_3 &= -20 \\ 3x_1 - 6x_2 + 5x_3 &= 23 \end{aligned}$	<p>6)</p> $\begin{aligned} 0,4x_1 + 0,8x_2 + 1,3x_3 &= 4,4 \\ 2,2x_1 - 1,4x_2 - 3,5x_3 &= -8,7 \\ -3x_1 - 1,5x_2 + x_3 &= -2,5 \end{aligned}$
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<p>7)</p> $\begin{aligned} x_1 - 2x_2 + 3x_3 + 4x_4 &= 8 \\ 2x_1 - 3x_2 + 4x_3 - 3x_4 &= 3 \\ 3x_2 + 4x_3 - x_4 &= 3 \\ x_1 + x_2 + x_3 + x_4 &= 3 \end{aligned}$	<p>8)</p> $\begin{aligned} 2x_1 + x_2 + x_3 + 2x_4 + 3x_5 &= 24 \\ x_1 + 2x_2 + 3x_3 + x_4 + x_5 &= 25 \\ 3x_1 + 2x_2 + 3x_3 + x_4 + 2x_5 &= 36 \\ x_1 + x_2 + 2x_3 + x_4 + x_5 &= 18 \\ x_1 + 2x_2 + 3x_3 + 3x_4 + 2x_5 &= 30 \end{aligned}$
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<p>9)</p> $\begin{aligned} x_1 - 3x_2 + 2x_3 &= 2 \\ 3x_2 - 2x_3 &= 1 \\ -6x_2 + 4x_3 &= 3 \end{aligned}$	<p>10)</p> $\begin{aligned} x_1 + 2x_2 - x_3 &= 2 \\ 2x_2 - 4x_3 &= 1 \\ 3x_2 - 6x_3 &= 1,5 \end{aligned}$	<p>11)</p> $\begin{aligned} x_1 - 4x_2 + x_3 &= 2 \\ 2x_2 - 4x_3 &= 6 \\ 3x_2 - 7x_3 &= 5 \end{aligned}$
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<p>12)</p> $\begin{aligned} x_1 + 2x_2 - x_3 &= 2 \\ x_1 + 2x_2 - 3x_3 &= 6 \\ -4x_3 &= 8 \end{aligned}$	<p>13)</p> $\begin{aligned} x_1 + x_2 + x_3 &= 3 \\ x_1 + 2x_2 + 3x_3 &= 6 \end{aligned}$	<p>14)</p> $\begin{aligned} -3x_1 + 6x_2 - 6x_3 &= 5 \\ 2x_1 - 4x_2 + 4x_3 &= -2 \end{aligned}$
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<p>15)</p> $\begin{aligned} -6x_1 - 3x_2 + 6x_3 &= 9 \\ 4x_1 + 2x_2 - 5x_3 &= -6 \end{aligned}$	<p>16)</p> $\begin{aligned} 3x_1 + 4x_2 + 2x_3 &= 5 \\ 2x_1 - 3x_2 + x_3 &= 8 \\ 2x_3 &= 6 \end{aligned}$	<p>17)</p> $\begin{aligned} 3x_1 + 2x_2 + 3x_3 &= 9 \\ 4x_2 - 3x_3 &= 6 \\ 2x_1 + 4x_2 &= 10 \end{aligned}$
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<p>18)</p> $\begin{aligned} 2x_1 - 3x_2 + 4x_3 &= 1 \\ 3x_1 + x_2 - 5x_3 &= 7 \\ 4x_1 + 5x_2 - 14x_3 &= 13 \end{aligned}$	<p>19)</p> $\begin{aligned} x_1 + x_3 &= 2 \\ x_2 + x_3 &= 4 \\ x_1 + x_2 &= 5 \\ x_1 + x_2 + x_3 &= 0 \end{aligned}$	<p>20)</p> $\begin{aligned} x_1 + x_2 + x_3 &= 15 \\ 2x_1 - x_2 + 7x_3 &= 50 \\ 3x_1 + 11x_2 - 9x_3 &= 1 \\ x_1 - x_2 + x_3 &= 5 \end{aligned}$
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<p>21)</p> $\begin{aligned} 7x_1 + 11x_2 + 13x_3 &= 0 \\ x_1 - x_2 - x_3 &= 1 \\ 2x_1 + 3x_2 + 4x_3 &= 0 \\ 9x_1 + 10x_2 + 11x_3 &= 0 \end{aligned}$	<p>22)</p> $\begin{aligned} x_1 + 2x_2 - 3x_3 + x_4 &= 0 \\ x_2 - x_4 &= 2 \\ 2x_1 + 3x_2 - 3x_3 + 5x_4 &= -3 \\ -x_1 + x_2 + 4x_3 &= 4 \end{aligned}$
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23)

$$\begin{aligned} 2x_3 - x_4 &= 1 \\ x_1 + x_2 + x_3 + x_4 &= 4 \\ 2x_1 + 2x_2 - 4x_3 + 5x_4 &= 5 \\ x_1 + x_2 - 7x_3 + 5x_4 &= 0 \end{aligned}$$

Lösungen

1)

x_1	x_2	x_3	b	Op	KS
1	3	-2	4,5	G1	6,5
-1	2	-3	1,5	G2	-0,5
3	-4	2	0,9	G3	1,9
1	3	-2	4,5	G4=G1	6,5
0	5	-5	6	G5=G1+G2	6
0	-13	8	-12,6	G6=-3*G1+G3	-17,6
-5	0	-5	-4,5	G7=-5G4+3G5	6,5
0	5	-5	6	G8=G5	-0,5
0	0	-25	15	G9=13*G5+5*G6	-10
25	0	0	37,5	G10=G9-5G7	62,5
0	-25	0	-15	G11=G9-5G8	-40
0	0	-25	15	G12=G9	-10
1	0	0	1,5	G13=G10/25	2,5
0	1	0	0,6	G14=G11/-25	1,6
0	0	1	-0,6	G15=G12/-25	0,4

$$L = \{(1,5; 0,6; -0,6)\}$$

2)

x_1	x_2	x_3	b	Op	KS
1,6	-0,5	2	0,1	G1	3,2
2	1,2	-1	1,8	G2	4
0,8	-2	-5	7,8	G3	1,6
16	-5	20	1	G4=10*G1	32
10	6	-5	9	G5=5*G2	20
4	-10	-25	39	G6=5*G3	8
16	-5	20	1	G7=G4	32
0	73	-140	67	G8=-5*G4+8*G5	0
0	35	120	-155	G9=G4-4*G6	0
16	-5	20	1	G10=G7	32
0	73	-140	67	G11=G8	0
0	7	24	-31	G12=G9/5	0
1168	0	760	408	G13=73*G10+5*G11	2336
0	73	-140	67	G14=G8	0
0	0	-2732	2732	G15=-73*G12+7*G11	0
146	0	95	51	G16=G13/8	292
0	73	-140	67	G17=G14	0
0	0	1	-1	G18=G15/-2732	0
146	0	0	146	G19=-95*G18+G16	292
0	73	0	-73	G20=140*G18+G17	0
0	0	1	-1	G21=G18	0
1	0	0	1	G22=G19/146	2
0	1	0	-1	G23=G20/73	0
0	0	1	-1	G24=G21	0

$L = \{(1; -1; -1)\}$

3)

x_1	x_2	x_3	b	Op	KS
0,4	0,8	1,2	1,8	G1	4,2
2,1	-1,4	-3,5	10,5	G2	7,7
-3	-2,5	1	-3,3	G3	-7,8
2	4	6	9	G4=5*G1	21
21	-14	-35	105	G5=10*G2	77
-30	-25	10	-33	G6=10*G3	-78
2	4	6	9	G7=G4	21
0	-112	-196	21	G8=-21*G4+2*G5	-287
0	35	100	102	G9=15*G4+G6	237
56	0	-28	273	G10=28*G7+G8	301
0	-112	-196	21	G11=G8	-287
0	0	4340	12159	G12=35*G8+112*G9	16499
243040	0	0	1525272	G13=4340*G10+28*G12	1768312
0	-486080	0	2474304	G14=4340*G11+196*G12	1988224
0	0	4340	12159	G15=G12	4340
1	0	0	1525272/243040	G16=G13/243040	
0	1	0	2474304/-486080	G17=G14/-486080	
0	0	1	12159/4340	G18=G15/4340	

$L = \{(3891/620; -789/155; 1737/620)\}$

4)

x_1	x_2	x_3	b	Op	KS
1	-0,5	2	-3	G1	-0,5
2	1,2	-1	4	G2	6,2
3	-2	2,5	-2	G3	1,5
2	-1	4	-6	G4=2*G1	-1
10	6	-5	20	G5=5*G2	31
6	-4	5	-4	G6=2*G3	3
2	-1	4	-16	G7=G4	-1
0	11	-25	50	G8=G5-5*G4	36
0	-1	-7	204	G9=G6-3*G4	6
22	0	19	-16	G10=11*G7+G8	25
0	11	-25	50	G11=G8	36
0	0	-102	204	G12=11*G9+G8	102
2244	0	0	2244	G13=102*G10+19*G12	4488
0	-1122	0	0	G14=-102*G11+25*G12	-1122
0	0	-102	204	G15=G12	102
1	0	0	1	G16=G13/2244	2
0	1	0	0	G17=G14/1122	1
0	0	1	-2	G18=G15/-102	-1

$$L = \{(1; 0; -2)\}$$

5)

x_1	x_2	x_3	b	Op	KS
2	5	2	-4	G1	5
-2	4	-5	-20	G2	-23
3	-6	5	23	G3	25
2	5	2	-4	G4=G1	5
0	9	-3	-24	G5=G1+G2	-18
0	-27	4	58	G6=-3*G1+2*G3	35
18	0	33	84	G7=9*G4-5*G5	135
0	9	-3	-24	G8=G5	-18
0	0	-5	-14	G9=3*G5+G6	-19
90	0	0	-42	G10=5*G7+33*G9	48
0	-45	0	78	G11=-5*G8+3*G9	33
0	0	-5	-14	G12=G9	-19
1	0	0	-21/45	G13=G10/90	48/90
0	1	0	-26/15	G14=G11/-45	-11/15
0	0	1	14/5	G15=G12/-5	19/5

$$L = \{(-21/45; -26/15; 14/5)\}$$

6)

x_1	x_2	x_3	b	Op	KS
0,4	0,8	1,3	4,4	G1	6,9
2,2	-1,4	-3,5	-8,7	G2	-11,4
-3	-1,5	1	-2,5	G3	-6
4	8	13	44	G4=10*G1	69
22	-14	-35	-87	G5=10*G2	-114
-6	-3	2	-5	G6=2*G3	-12
4	8	13	44	G7=G4	69
0	-116	-213	-658	G8=11*G4+2*G5	-987
0	18	43	122	G9=3*G4+2*G6	183
116	0	-49	-40	G10=29*G7+2*G8	27
0	-116	-213	-658	G11=G8	-987
0	0	577	1154	G12=9*G8+58*G9	1731
66932	0	0	33466	G13=577*G10+49*G12	100398
0	-66932	0	-133864	G14=577*G11+213*G12	-200796
0	0	577	1154	G15=G12	1731
1	0	0	0,5	G16=G13/66932	1,5
9	1	0	-2	G17=G14/-66932	8
0	0	1	2	G18=G15/577	3

$$L = \{(0,5; -2; 2)\}$$

7)

x_1	x_2	x_3	x_4	b	Op	KS
1	-2	3	4	8	G1	14
2	-3	4	-3	3	G2	3
0	3	4	-1	3	G3	9
1	1	1	1	3	G4	7
1	-2	3	4	8	G5=G1	14
0	1	-2	-19	-13	G6=-2*G1+G2	-25
0	3	4	-1	3	G7=G3	9
0	3	-2	-3	-5	G8=-G1+G4	-7
1	0	-1	-18	-18	G9=G5+2*G6	-36
0	1	-2	-11	-13	G10=G6	-25
0	0	10	32	42	G11=-3*G6+G7	84
0	0	4	30	34	G12=-3*G6+G8	68
10	0	0	-148	-138	G13=10*G9+G11	-276
0	5	0	-23	-23	G14=5*G10+G11	-41
0	0	10	32	42	G15=G11	84
0	0	0	86	86	G16=-2*G11+5*G12	172
860	0	0	0	860	G17=148*G16+86*G13	1720
0	430	0	0	0	G18=23*G16+86*G14	430
0	0	-430	0	-430	G19=-43*G15+16*G16	-860
0	0	0	86	86	G20=G16	172
1	0	0	0	1	G21=G17/860	2
0	1	0	0	0	G22=G18/430	1
0	0	1	0	1	G23=G19/-430	2
0	0	0	1	1	G24=G20/86	2

$$L = \{(1; 0; 1; 1)\}$$

8)

x_1	x_2	x_3	x_4	x_5	b	Op	KS
2	1	1	2	3	24	G1	33
1	2	3	1	1	25	G2	33
3	2	3	1	2	36	G3	47
1	1	2	1	1	18	G4	24
1	2	3	3	2	30	G5	41
2	1	1	2	3	24	G6=G1	33
0	-3	-5	0	1	-26	G7=-2*G2+G1	-33
0	-1	-3	4	5	0	G8=-2*G3+3G1	5
0	-1	-3	0	1	-12	G9=-2*G4+G1	-15
0	-3	-5	-4	-1	-36	G10=-2*G5+G1	-49
6	0	-2	6	10	46	G11=3G6+G7	66
0	-3	-5	0	1	-26	G12=G7	-33
0	0	4	-12	-14	-26	G13=-3*G8+G7	-48
0	0	4	0	-2	10	G14=-3*G9+G7	12
0	0	0	4	2	10	G15=-1*G10+G7	16
12	0	0	0	6	66	G16=2*G11+G13	84
0	-12	0	-60	-66	-234	G17=4*G12+5G13	-372
0	0	4	-12	-14	-26	G18=G13	-48
0	0	0	-12	-12	-36	G19=-1*G14+G13	-60
0	0	0	4	2	10	G20=G15	16
12	0	0	0	6	66	G21=G16	84
0	-12	0	0	-6	-54	G22=G17-5*G19	-72
0	0	-4	0	2	-10	G23=-1*G18+G19	-12
0	0	0	-12	-12	-36	G24=G19	-60
0	0	0	0	-6	-6	G25=3*G20+G19	-12
12	0	0	0	0	60	G26=G21+G25	72
0	12	0	0	0	48	G27=-1*G22+G25	60
0	0	-12	0	0	-36	G28=3*G23+G25	-48
0	0	0	-12	0	-24	G29=G24-2G25	-36
0	0	0	0	-6	-6	G30=G25	-12
1	0	0	0	0	5	G31=G26/12	6
0	1	0	0	0	4	G32=G27/12	5
0	0	1	0	0	3	G33=G28/-12	4
0	0	0	1	0	2	G34=G29/-12	3
0	0	0	0	1	1	G35=G30/-6	2

$L = \{(5; 4; 3; 2; 1)\}$

9)

x_1	x_2	x_3	b	Op	KS
1	-3	2	2	G1	2
0	3	-2	1	G2	2
0	-6	4	3	G3	1
1	-3	2	2	G4=G1	2
0	3	-2	1	G5=G2	2
0	0	0	5	G6=2*G2+G3	5

$L = \{ \}$

10)

x_1	x_2	x_3	b	Op	KS
1	2	-1	2	G1	4
0	2	-4	1	G2	-1
0	3	-6	1,5	G3	-1,5
-1	0	-3	-1	G4=G2-G1	-5
0	2	-4	1	G5=G2	-1
0	0	0	0	G6=-1,5*G2+G3	0
1	0	3	1	G7=G4/-1	5
0	1	-2	0,5	G8=G5/2	-0,5

$L = \{ (1-3r; 1/2+2r) \mid r \in \mathbb{R} \}$

11)

x_1	x_2	x_3	b	Op	KS
1	-4	1	2	G1	0
0	2	-4	6	G2	4
0	3	-7	5	G3	1
1	0	-7	14	G4=G1+2G2	8
0	2	-4	6	G5=G2	4
0	0	-1	-4	G6=-1,5*G2+G3	-5
1	0	0	42	G7=-7*G6+G4	43
0	2	0	22	G8=-4*G6+G5	24
0	0	-1	-4	G9=G6	-5
1	0	0	42	G10=G7	43
0	1	0	11	G11=G8/2	12
0	0	1	4	G12=G9/-1	5

$L = \{ (40, 11, 4) \}$

12)

x_1	x_2	x_3	b	Op	KS
1	2	-1	2	G1	4
1	2	-3	6	G2	6
0	0	-4	8	G3	4
1	2	-1	2	G4=G1	4
0	0	-2	4	G5=-G1+G2	2
0	0	-4	8	G6=G3	4
-2	-4	0	0	G7=-2*G4+G5	-6
0	0	-2	4	G8=G5	2
0	0	0	0	G9=-2*G5+G6	0
1	2	0	0	G10=G7/-2	3
0	0	1	-2	G11=G8/-2	-1

$$L = \{(-2r; r; -2) \mid r \in \mathbb{R}\}$$

13)

x_1	x_2	x_3	b	Op	KS
1	1	1	3	G1	6
1	2	3	6	G2	12
1	1	1	3	G3=G1	5
0	1	2	3	G4=-G1+G2	6
-1	0	1	0	G5=-G3+G4	0
0	1	2	3	G6=G4	6
1	0	-1	0	G7=G5/-1	0
0	1	2	3	G8=G6	6

$$L = \{(r; 3-2r; r) \mid r \in \mathbb{R}\}$$

14)

x_1	x_2	x_3	b	Op	KS
-3	6	-6	5	G1	6
2	-4	4	-2	G2	12
-3	6	-6	5	G3=G1	5
0	0	0	4/3	G4=2/3*G1+G2	6

$$L = \{\}$$

15)

x_1	x_2	x_3	b	Op	KS
-6	-3	6	9	G1	6
4	2	-5	-6	G2	-5
-6	-3	6	9	G3=G1	6
0	0	-1	0	G4=2/3*G1+G2	-1
-6	-3	0	9	G5=6*G4+G3	0
0	0	-1	0	G6=G4	-1
1	0,5	0	-1,5	G7=G5/-6	0
0	0	1	0	G8=G6/-1	1

$$L = \{(-1,5 \ -0,5r; \ r; \ 0) \mid r \in \mathbb{R}\}$$

16)

x_1	x_2	x_3	b	Op	KS
3	4	2	5	G1	14
2	-3	1	8	G2	8
0	0	2	6	G3	8
3	4	2	5	G4=G1	14
0	17	1	-14	G5=-3*G2+2*G1	4
0	0	2	6	G6=G3	8
51	0	30	141	G7=17*G4-4*G5	222
0	17	1	-14	G8=G5	4
0	0	2	6	G9=G6	8
51	0	0	51	G10=-15*G9+G7	102
0	-34	0	34	G11=-2*G5+G9	0
0	0	2	6	G12=G9	8
1	0	0	1	G13=G10/51	2
0	1	0	-1	G14=G11/-34	0
0	0	1	3	G15=G12/2	4

$$L = \{(1; \ -1; \ 3)\}$$

17)

x_1	x_2	x_3	b	Op	KS
3	2	3	9	G1	17
0	4	-3	6	G2	7
2	4	0	10	G3	16
3	2	3	9	G4=G1	14
0	4	-3	6	G5=G2	7
0	-8	6	-12	G6=-3*G3+2*G1	-14
3	6	0	15	G7=G4	24
0	4	-3	6	G8=G5	7
0	0	0	0	G9=2*G5+G6	0
1	2	0	5	G10=G7/3	8
0	-4/3	1	-2	G11=G8/-3	-10/3

$$L = \{(5-2r; r; -2+4/3*r) \mid r \in \mathbb{R}\}$$

18)

x_1	x_2	x_3	b	Op	KS
2	-3	4	1	G1	4
3	1	-5	7	G2	8
4	5	-14	13	G3	8
2	-3	4	1	G4=G1	4
0	11	-22	11	G5=-3*G1+2*G2	0
0	11	-22	11	G6=-2*G1+G3	0
22	0	-22	44	G7=11*G4+3*G5	44
0	11	-22	11	G8=G5	0
0	0	0	0	G9=G5-G6	0
1	0	-1	2	G10=G7/22	2
0	1	-2	1	G11=G8/11	0

$$L = \{(2+r; 1+2r; r) \mid r \in \mathbb{R}\}$$

19)

x_1	x_2	x_3	b	Op	KS
1	0	1	2	G1	4
0	1	1	4	G2	6
1	1	0	5	G3	7
1	1	1	0	G4	3
1	0	1	2	G5=G1	4
0	1	1	4	G6=G2	6
0	1	-1	3	G7=-G1+G3	3
0	1	0	-2	G8=-G1+G4	-1
1	0	1	2	G9=G5	4
0	1	1	4	G10=G6	6
0	0	-2	-1	G11=-G6+G7	-3
0	0	-1	-6	G12=-G6+G8	-7
1	0	1	2	G13=G9	4
0	1	1	4	G14=G10	6
0	0	-2	-1	G15=G11	-3
0	0	0	11	G16=-2*G12+G11	11

$L = \{ \}$

20)

x_1	x_2	x_3	b	Op	KS
1	1	1	15	G1	18
2	-1	7	50	G2	58
3	11	-9	1	G3	6
1	-1	1	5	G4	6
1	1	1	15	G5=G1	18
0	-3	5	20	G6=-2*G1+G2	22
0	8	-12	-44	G7=-3*G1+G3	-48
0	-2	0	-10	G8=-G1+G4	-12
3	0	8	65	G9=3*G5+G6	76
0	-3	5	20	G10=G6	22
0	0	4	28	G11=8*G6+3*G7	32
0	0	-10	-70	G12=-2*G6+3*G8	-80
3	0	0	9	G13=-2*G11+G9	12
0	12	0	60	G14=-4*G10+5*G11	72
0	0	4	28	G15=G11	32
0	0	0	0	G16=2*G12+5*G11	0
1	0	0	3	G17=G13/3	4
0	1	0	5	G18=G14/12	6
0	0	1	7	G19=G15/4	8

$L = \{ (3; 5; 7) \}$

21)

x_1	x_2	x_3	b	Op	KS
7	11	13	0	G1	31
1	-1	-1	1	G2	0
2	3	4	0	G3	9
9	10	11	0	G4	30
7	11	13	0	G5=G1	31
0	18	20	-7	G6=-7*G2+G1	31
0	-1	2	0	G7=-2*G1+7*G3	1
0	-29	-40	0	G8=-9*G1+7*G4	-69
7	11	13	0	G9=G5	31
0	18	20	-7	G10=G6	31
0	0	56	-7	G11=G6+18*G7	49
0	0	-140	-203	G12=29*G6+18*G8	-343
7	11	13	0	G13=G9	31
0	18	20	-7	G14=G10	31
0	0	56	-7	G15=G11	49
0	0	0	-882	G16=10*G11+4*G12	-882

$L = \{ \}$

22)

x_1	x_2	x_3	x_4	b	Op	KS
1	2	-3	1	0	G1	1
0	1	0	-1	2	G2	2
2	3	-3	5	-3	G3	4
-1	1	4	0	4	G4	8
1	2	-3	1	0	G5=G1	1
0	1	0	-1	2	G6=G2	2
0	-1	3	3	-3	G7=-2*G1+G3	2
0	3	1	1	4	G8=G1+G4	9
1	0	-3	3	-4	G9=-2*G6+G5	-3
0	1	0	-1	2	G10=G6	2
0	0	3	2	-1	G11=G6+G7	4
0	0	1	4	-2	G12=-3*G6+G8	3
1	0	0	5	-5	G13=G9+G11	1
0	1	0	-1	2	G14=G10	2
0	0	3	2	-1	G15=G11	4
0	0	0	-10	5	G16=G11-3*G12	-5
2	0	0	0	-5	G17=G16+2*G13	-3
0	-10	0	0	-15	G18=G16-10*G14	-25
0	0	15	0	0	G19=G16+5*G15	15
0	0	0	-10	5	G20=G16	-5
1	0	0	0	-2,5	G21=G17/2	-1,5
0	1	0	0	1,5	G22=G18/-10	2,5
0	0	1	0	0	G23=G19/15	1
0	0	0	1	-0,5	G24=G20/-10	0,5

$L = \{ (-2,5; 1,5; 0; -0,5) \}$

23)

x_1	x_2	x_3	x_4	b	Op	KS
0	0	2	-1	1	G1	2
1	1	1	1	4	G2	8
2	2	-4	5	5	G3	10
1	1	-7	5	0	G4	0
0	0	2	-1	1	G5=G1	2
-2	-2	0	-3	-7	G6=-2*G2+G1	-14
2	2	0	3	7	G7=2*G1+G3	14
2	2	0	3	7	G8=7*G1+2*G4	14
0	0	2	-1	1	G9=G5	2
-2	-2	0	-3	-7	G10=G6	-14
0	0	0	0	0	G11=G6+G7	0
0	0	0	0	0	G12=G6+G8	0
0	0	2	-1	1	G13=G9	2
-2	-2	0	-3	-7	G14=G10	-14
0	0	1	-0,5	0,5	G15=-G13/2	1
1	1	0	1,5	3,5	G16=G14/-2	7

setze:

$$x_2 = s, \quad x_4 = t$$

also:

$$x_1 = 3,5 - s - 1,5t$$

$$x_3 = 0,5 + 0,5s$$

$$L = \{(3,5-s-1,5t; s; 0,5+0,5t) \mid s \in \mathbb{R} \wedge t \in \mathbb{R}\}$$