

1 ÜBUNGSAUFGABEN MESK 2BK11

Bemerkungen zu einigen mathematischen Notationen:

$x \leq 17$ ist eine mathematische Kurzschreibweise für: $x < 17 \vee x = 17$

$x \geq 17$ ist eine mathematische Kurzschreibweise für: $x > 17 \vee x = 17$

$15 \leq x < 19$ ist eine mathematische Kurzschreibweise für: $15 \leq x \wedge x < 19$

1) Geben Sie sämtliche Teilmengen von $A = \{7; 8; 9\}$ an.

2) Bestimmen Sie von folgenden Mengen A, B, C die Durchschnitte

$A \cap B, A \cap C, B \cap C$:

a) $A = \{13; 14; 15; 16\}, B = \{12; 13; 14\}, C = \{15; 16; 17\}$

b) $A = \{x \in \mathbb{N} \mid x < 10\}, B = \{x \in \mathbb{N} \mid x > 5\}, C = \{x \in \mathbb{N} \mid x = 10\}$

c) $A = \{x \in \mathbb{N} \mid x > 40\}, B = \{x \in \mathbb{N} \mid x > 50\}, C = \{x \in \mathbb{N} \mid 30 \leq x \leq 50\}$

3) Bestimmen Sie von folgenden Mengen A, B, C die Vereinigungsmengen

$A \cup B, A \cup C, B \cup C$:

a) $A = \{9; 12\}, B = \{6; 9; 12\}, C = \{3; 15\}$

b) $A = \{x \in \mathbb{N} \mid 12 < x < 15\}, B = \{x \in \mathbb{N} \mid 15 \leq x \leq 18\}, C = \{x \in \mathbb{N} \mid 17 < x \leq 19\}$

c) $A = \{x \in \mathbb{N} \mid x = 77\}, B = \{x \in \mathbb{N} \mid 74 < x < 77\}, C = \{x \in \mathbb{N} \mid 77 \leq x \leq 79\}$

4) Bestimmen Sie von den Mengen A und B die Mengen $A \setminus B$ und $B \setminus A$:

a) $A = \{1; 3; 9; 12\}, B = \{6; 12; 18\}$

b) $A = \{1; 3; 5\}, B = \{2; 4; 6\}$

c) $A = \{x \in \mathbb{N} \mid x < 5\}, B = \{x \in \mathbb{N} \mid x < 8\}$

d) $A = \{x \in \mathbb{N} \mid x \leq 4\}, B = \{x \in \mathbb{N} \mid 2 < x < 7\}$

5) Bestimmen Sie die Vereinigungsmengen von:

a) $A = \{x \in \mathbb{R} \mid -1 < x < 1\}, B = \{x \in \mathbb{R} \mid 1 \leq x < 2\}$

b) $A = \{x \in \mathbb{R} \mid 3 \leq x \leq 8\}, B = \{x \in \mathbb{R} \mid -2 \leq x < 5\}$

c) $A = \{x \in \mathbb{R} \mid -3 < x < -2\}, B = \{x \in \mathbb{R} \mid 5 < x \leq 9\}$

6) Bestimmen Sie die Schnittmengen von:

a) $E = \{x \in \mathbb{R} \mid -1 < x < 1\}, F = \{x \in \mathbb{R} \mid 1 \leq x < 2\}$

b) $E = \{x \in \mathbb{R} \mid 4 < x < 7\}, F = \{x \in \mathbb{R} \mid 5 \leq x \leq 9\}$

7) Bestimmen Sie:

a) $X \cup X$ b) $Z \cap Z$ c) $A \setminus A$ d) $B \cup \emptyset$ e) $B \cap \emptyset$

8) Es sei $A \subset B$. Bestimmen Sie:

a) $A \cup B$ b) $A \cap B$ c) $A \setminus B$

9) Bestimmen Sie von folgenden Mengen

$A = \{4; 5; 6\}, B = \{5; 6; 7; 9\}, C = \{7; 8\}$

die Mengen:

a) $A \cup B$ b) $A \cup C$ c) $B \cap C$ d) $A \cup (B \cap C)$ e) $(A \cup B) \cap (A \cup C)$

10) Welche der Aussagen sind richtig bzw. falsch ?

a) $\{3\} \subset \{3; 4\}$ b) $3 \subset \{3; 4\}$ c) $3 \subset \{3\}$ d) $\{3; 4\} \subset \{3; 7; 4\}$

11) Welche der Aussagen sind richtig bzw. falsch ?

Falls eine Aussage falsch ist, geben Sie ein Element an, das Element der einen Menge (z.B. der Menge LM links des Gleichheitszeichens), aber nicht Element der anderen Menge (z.B. der Menge RM rechts des Gleichheitszeichens) ist.

a) $\{x \in \mathbb{R} \mid x > 49\} = \{x \in \mathbb{R} \mid x \geq 50\}$

b) $\{x \in \mathbb{N} \mid x > 49\} = \{x \in \mathbb{N} \mid x \geq 50\}$

c) $\{x \in \mathbb{N} \mid x > 49\} = \{x \in \mathbb{R} \mid x \geq 50\}$

d) $\{x \in \mathbb{N} \mid x > 49\} \subset \{x \in \mathbb{R} \mid x \geq 50\}$

e) $\{x \in \mathbb{N} \mid x < 4\} = \{x \in \mathbb{N} \mid x \leq 3\}$

f) $\{x \in \mathbb{R} \mid x < 4\} = \{x \in \mathbb{R} \mid x \leq 3\}$

Lösungen:

1) $\{7;8;9\}$, $\{7;8\}$, $\{7;9\}$, $\{8;9\}$, $\{7\}$, $\{8\}$, $\{9\}$, $\{\}$

2)

a) $A \cap B = \{13; 14\}$, $A \cap C = \{15; 16\}$, $B \cap C = \emptyset$

b) $A \cap B = \{x \in \mathbb{N} \mid 6 \leq x \leq 9\}$, $A \cap C = \emptyset$, $B \cap C = \{10\}$

c) $A \cap B = \{x \in \mathbb{N} \mid x \geq 51\}$, $A \cap C = \{x \in \mathbb{N} \mid 41 \leq x \leq 50\}$, $B \cap C = \emptyset$

3)

a) $A \cup B = \{6; 9; 12\}$, $A \cup C = \{3; 9; 12; 15\}$, $B \cup C = \{3; 6; 9; 12; 15\}$

b) $A \cup B = \{x \in \mathbb{N} \mid 13 \leq x \leq 18\}$, $A \cup C = \{13; 14; 18; 19\}$,

$B \cup C = \{x \in \mathbb{N} \mid 15 \leq x \leq 19\}$

c) $A \cup B = \{x \in \mathbb{N} \mid 75 \leq x \leq 77\}$, $A \cup C = \{x \in \mathbb{N} \mid 77 \leq x \leq 79\}$,

$B \cup C = \{x \in \mathbb{N} \mid 75 \leq x \leq 79\}$

4)

a) $A \setminus B = \{1; 3; 9\}$, $B \setminus A = \{6; 18\}$

b) $A \setminus B = \{1; 3; 5\}$, $B \setminus A = \{2; 4; 6\}$

c) $A \setminus B = \emptyset$, $B \setminus A = \{5; 6; 7\}$

d) $A \setminus B = \{x \in \mathbb{N} \mid x \leq 2\}$, $B \setminus A = \{5; 6\}$

5)

a) $A \cup B = \{x \in \mathbb{R} \mid -1 < x < 2\}$ b) $A \cup B = \{x \in \mathbb{R} \mid -2 \leq x \leq 8\}$

c) $A = \{x \in \mathbb{R} \mid -3 < x < -2\}$, $B = \{x \in \mathbb{R} \mid 5 < x \leq 9\}$

$A \cup B = \{x \in \mathbb{R} \mid -3 < x < -2 \vee 5 < x \leq 9\}$

6)

a) $E \cap F = \emptyset$ b) $E \cap F = \{x \in \mathbb{R} \mid 5 \leq x < 7\}$

7)

a) X b) Z c) \emptyset d) B e) \emptyset

8)

a) B b) A c) \emptyset

9)

a) $\{4;5;6;7;9\}$ b) $\{4;5;6;7;8\}$ c) $\{7\}$ d) $\{4;5;6;7\}$ e) $\{4;5;6;7\}$

10)

a) richtig b) falsch c) falsch d) richtig

11)

a) falsch ($49,1 \in LM$ und $49,1 \notin RM$)

b) richtig

c) falsch ($50,7 \in RM$ und $50,7 \notin LM$)

d) richtig

e) richtig

f) falsch ($3,9 \in LM$ und $3,9 \notin RM$)

2 ÜBUNGSAUFGABEN MESK 2BK11

Formen Sie die Terme so in **einfachere** Terme um, dass sich allgemeingültige Gleichungen ergeben.

Beispiel: $8a + 7a = 15a$

- | | |
|---|--|
| <p>1) $17a - 23b + 35c - 9a - 41c + 30b$</p> <p>3) $(-\frac{1}{2}ab) \cdot (+\frac{8}{3}bc) \cdot (-\frac{3}{4}ac)$</p> <p>5) $6x - [2y - (4z + (3x - 2y) + 2x) - 5z]$</p> <p>7) $(\frac{2}{3}u - \frac{4}{5}v + \frac{1}{2}w) \cdot (-\frac{3}{4}v)$</p> <p>9) $a(2b - c) + 3b(2a + c) - 5c(2a - b)$</p> <p>11) $\frac{20a^2 + 15ab - 35ac}{5a}$</p> <p>13) $\frac{ax - ay}{5x - 5y}$</p> <p>15) $\frac{\frac{a}{b} + 1}{\frac{b}{a} + 1}$</p> <p>17) $1 + \frac{1}{x + y}$</p> <p>19) $\frac{9a + 8b - 3c}{2ab} + \frac{3b - 4a - 12c}{3ac} - \frac{27c - 7a - 8b}{6bc}$</p> <p>20) $\frac{2a}{3x} - \frac{2a}{3x - 2y}$</p> <p>22) $\frac{34a^3b^2}{(+2a) \cdot (-b)} - \frac{54a^4b^3}{(-a^2) \cdot (9b^2)}$</p> <p>24) $3(x + y + z) - 5(x + y - z) - 2(y - x - z)$</p> | <p>2) $4 \cdot a \cdot 2 \cdot c \cdot 5 \cdot b \cdot 3 \cdot c \cdot 2 \cdot a$</p> <p>4) $(+\frac{3}{7}x) : (-x)$</p> <p>6) $(x - 2y - 6z) \cdot (-4a)$</p> <p>8) $x^2 - 8x + 16$</p> <p>10) $-120t - 48t^2 - 75$</p> <p>12) $\frac{38u^2 - 57uv}{2u - 3v}$</p> <p>14) $\frac{m^4 - n^4}{m^2 + n^2}$
$\frac{x^2 - y^2}{2y - 2x}$</p> <p>16) $\frac{\frac{2}{x} - \frac{2}{y}}{\frac{2}{x} - \frac{2}{y}}$</p> <p>18) $\frac{2x + 1}{x} - \frac{x + 1}{x}$</p> <p>21) $\frac{3(1 - 2a)}{c(b - 1)} \cdot \frac{4(2a - 1)}{(1 + b) \cdot c}$</p> <p>23) $(\frac{2z}{3xy} + \frac{5x}{6yz} - \frac{6y}{7xz}) \cdot 42xyz$</p> |
|---|--|

Tipps zur Herstellung eigener Übungsaufgaben:

Nehmen Sie eine binomische Formel, wie z.B:

$$(3s - 5)$$

Multiplizieren Sie diese Formel aus:

$$9s^2 - 30s + 25$$

So, und nun legen Sie die obige Formel $(3s - 5)$ weg und versuchen die Formel:

$$9s^2 - 30s + 25$$

als Produkt zu schreiben.

Lösungen:

$$1) 17a - 23b + 35c - 9a - 41c + 30b = 8a + 7b - 6c$$

$$2) 4 \cdot a \cdot 2 \cdot c \cdot 5 \cdot b \cdot 3 \cdot c \cdot 2 \cdot a = 240a^2bc^2$$

$$3) \left(-\frac{1}{2}ab\right) \cdot \left(+\frac{8}{3}bc\right) \cdot \left(-\frac{3}{4}ac\right) = \left(-\frac{1}{2}\right) \cdot \left(+\frac{8}{3}\right) \cdot \left(-\frac{3}{4}\right) \cdot a \cdot b \cdot b \cdot c \cdot a \cdot c = a^2b^2c^2$$

$$4) \left(+\frac{3}{7}x\right) : (-x) = -\frac{3}{7}$$

$$5) 6x - [2y - (4z + (3x - 2y) + 2x) - 5z] = 6x - 2y + (4z + (3x - 2y) + 2x) + 5z \\ = 6x - 2y + 4z + (3x - 2y) + 2x + 5z = 6x - 2y + 4z + 3x - 2y + 2x + 5z = 11x - 4y + 9z$$

$$6) (x - 2y - 6z) \cdot (-4a) = x \cdot (-4a) - 2y \cdot (-4a) - 6z \cdot (-4a) = -4ax + 8ay + 24az$$

$$7) \left(\frac{2}{3}u - \frac{4}{5}v + \frac{1}{2}w\right) \cdot \left(-\frac{3}{4}v\right) = \frac{2}{3}u \cdot \left(-\frac{3}{4}v\right) - \frac{4}{5}v \cdot \left(-\frac{3}{4}v\right) + \frac{1}{2}w \cdot \left(-\frac{3}{4}v\right) \\ = -\frac{1}{2}uv + \frac{3}{5}v^2 - \frac{3}{8}vw$$

$$8) x^2 - 8x + 16 = x^2 - 2 \cdot 4 \cdot x + 4^2 = (x - 4)^2$$

$$9) a(2b - c) + 3b(2a + c) - 5c(2a - b) = 2ab - ac + 6ab + 3bc - 10ac + 5bc \\ = 8ab - 11ac + 8bc$$

$$10) -120t - 48t^2 - 75 = -3(40t + 16t^2 + 25) = -3((4t)^2 + 2 \cdot 4t \cdot 5 + 5^2) = -3(4t + 5)^2$$

$$11) 1. \text{ Lösung: } \frac{20a^2 + 15ab - 35ac}{5a} = \frac{20a^2}{5a} + \frac{15ab}{5a} - \frac{35ac}{5a} = 4a + 3b - 7c$$

$$2. \text{ Lösung: } \frac{20a^2 + 15ab - 35ac}{5a} = (20a^2 + 15ab - 35ac) : 5a = 4a + 3b - 7c$$

$$3. \text{ Lösung: } \frac{20a^2 + 15ab - 35ac}{5a} = \frac{5a(4a + 3b - 7c)}{5a} = 4a + 3b - 7c$$

$$12) \frac{38u^2 - 57uv}{2u - 3v} = \frac{19u(2u - 3v)}{2u - 3v} = 19u$$

$$13) \frac{ax - ay}{5x - 5y} = \frac{a(x - y)}{5(x - y)} = \frac{a}{5}$$

$$14) \frac{m^4 - n^4}{m^2 + n^2} = \frac{(m^2)^2 - (n^2)^2}{m^2 + n^2} = \frac{(m^2 - n^2)(m^2 + n^2)}{m^2 + n^2} = m^2 - n^2$$

$$15) \frac{\frac{a}{b} + 1}{\frac{b}{a} + 1} = \frac{\frac{a}{b} + \frac{b}{b}}{\frac{b}{a} + \frac{a}{a}} = \frac{\frac{a+b}{b}}{\frac{b+a}{a}} = \frac{(a+b)a}{b(a+b)} = \frac{a}{b}$$

$$16) \frac{\frac{x^2}{2} - \frac{y^2}{2}}{\frac{x}{x} - \frac{y}{y}} = \frac{\frac{2xy}{2y} - \frac{2xy}{2x}}{\frac{2y-2x}{xy}} = \frac{\frac{x^3 - y^3}{2xy}}{\frac{2y-2x}{xy}} = \frac{(x^3 - y^3)xy}{2xy(2y-2x)} = \frac{x^3 - y^3}{4(y-x)}$$

$$17) 1 + \frac{1}{x+y} = \frac{x+y}{x+y} + \frac{1}{x+y} = \frac{x+y+1}{x+y}$$

$$18) \frac{2x+1}{x} - \frac{x+1}{x} = \frac{2x+1-(x+1)}{x} = \frac{2x+1-x-1}{x} = 1$$

$$19) \frac{9a+8b-3c}{2ab} + \frac{3b-4a-12c}{3ac} - \frac{27c-7a-8b}{6bc} =$$

$$\frac{(9a+8b-3c) \cdot 3c + (3b-4a-12c) \cdot 2b - (27c-7a-8b) \cdot a}{6abc} =$$

$$\frac{27ac+24bc-9c^2+6b^2-8ab-24cb-27ac+7a^2+8ab}{6abc} =$$

$$\frac{7a^2+6b^2-9c^2}{6abc}$$

$$20) \frac{2a}{3x} - \frac{2a}{3x-2y} = \frac{(3x-2y)2a}{3x(3x-2y)} - \frac{2a \cdot 3x}{3x(3x-2y)} = \frac{(3x-2y)2a - 2a \cdot 3x}{3x(3x-2y)} = \frac{6ax - 4ay - 6ax}{3x(3x-2y)}$$

$$= \frac{-4ay}{3x(3x-2y)}$$

$$21) \frac{3(1-2a)}{c(b-1)} : \frac{4(2a-1)}{(1+b) \cdot c} = \frac{3(1-2a) \cdot (1+b) \cdot c}{c(b-1) \cdot 4(2a-1)} = \frac{3(1-2a) \cdot (1+b) \cdot c}{c \cdot -(1-b) \cdot 4 \cdot -(1-2a)} = \frac{3(1+b)}{4(1-b)}$$

$$22) \frac{34a^3b^2}{(+2a) \cdot (-b)} - \frac{54a^4b^3}{(-a^2) \cdot (9b^2)} = -17a^2b + 6a^2b = -11a^2b$$

$$23) \left(\frac{2z}{3xy} + \frac{5x}{6yz} - \frac{6y}{7xz} \right) \cdot 42xyz = \frac{2z}{3xy} \cdot 42xyz + \frac{5x}{6yz} \cdot 42xyz - \frac{6y}{7xz} \cdot 42xyz$$

$$= 28z^2 + 35x^2 - 36y^2$$

$$24) 3(x+y+z) - 5(x+y-z) - 2(y-x-z) =$$

$$3x+3y+3z-5x-5y+5z-2y+2x+2z = -4y+10z$$

3 Weitere ÜBUNGSAUFGABEN zur Termumformung MESK 2BK11

Formen Sie die Terme so um, daß sich allgemeingültige Gleichungen ergeben.

- | | | | |
|-----|---|-----|--|
| 1) | $2d+(5d+4e)-(10d-8e-38)-6d-17$ | 23) | $\frac{a}{y} : \frac{b}{y}$ |
| 2) | $7-\{[(26x+37y-25z)+19y-16a]-8x+9z+6\}$ | 24) | $\frac{4ax}{a-x} : \frac{8ay}{a-x}$ |
| 3) | $(3a+4x)-\{6a-[5x-(9a-8x)]+13a\}$ | 25) | $\left(a - \frac{x}{y}\right) : \frac{a}{y}$ |
| 4) | $4x+6y-\{6x-[7y-(5x+3y)]-(6y-8x)-3x\}$ | 26) | $(x-3)^2$ |
| 5) | $4xy-8xz-\{2yz-[3xy-(5xz+6yz)+7yz]-2xy\}$ | 27) | $(7n+8m)^2$ |
| 6) | $\{[4c-(5cd+d)]-[7d-(c-2cd)]\}-(c+d)$ | 28) | $(3x^2-4x^3)^2$ |
| 7) | $(-5i)\cdot(18a+12b-14c+15x)$ | 29) | $(y-3)(y+3)$ |
| 8) | $(y-9)\cdot(x-4)$ | 30) | $(3n+m)^3$ |
| 9) | $(a+b)(4x-5y)-(a-b)(5x+3y)$ | 31) | $(5m+4)(5m-4)$ |
| 10) | $(3a-5b)(6x-7y+9z)-(5x-8y+8z)(4a-5b)$ | 32) | $(-x-y)^2$ |
| 11) | $(4a+3b-5c)\cdot 7x-[(5a-4b+6c)\cdot 3x]$ | 33) | $(5m+7n)^2-(3m-4n)^2$ |
| 12) | $(4+2a-3c)\cdot(12-2d-5b)-[(12d-6b)\cdot 9a]$ | 34) | $(3x-7y)^2-(7x-3y)^2-(7x-3y)(3y+7x)$ |
| 13) | $(3a-2b)(2c-4d)(5x-2y)$ | 35) | $\frac{a^2+b^2}{a^2-b^2} \cdot \frac{a-b}{a+b}$ |
| 14) | $21abx-6by+15bz$ | 36) | $\frac{(a+b)^2}{a^2-b^2} \cdot \frac{a-b}{a+b}$ |
| 15) | $25ab+125ac+75ax$ | 37) | $\frac{4c^2-1}{d^2-4} \cdot \frac{d+2}{2c-1}$ |
| 16) | $axnd-axnc+abnd-abnc$ | 38) | $\frac{x^2+1+2x}{x^2+1} \cdot \frac{(x-1)^2}{(x+1)^2}$ |
| 17) | $R_{20}+R_{20}\cdot\alpha\cdot\Delta\vartheta+R_{20}\cdot\beta\cdot(\Delta\vartheta)^2$ | 39) | $\frac{(2x-6y)^2}{(x-y)^2} : \frac{x-3y}{x^2-y^2}$ |
| 18) | $\frac{a+b}{5x} + \frac{6a-5b}{5x} - \frac{2a+b}{5x}$ | 40) | $\frac{x^2-x^2y^2}{x^2+x^2y} : \frac{x+xy}{x^2+x^2y}$ |
| 19) | $\frac{a}{3x} - \frac{2a}{2x} + \frac{3a}{x}$ | 41) | $\frac{p^2-4p}{p^2-4} \cdot \frac{p+2}{p}$ |
| 20) | $\frac{6b+2}{4y-8} + \frac{5b-3}{2y-4} - \frac{3b-1}{y-2}$ | | |
| 21) | $\left(\frac{a}{x} - \frac{b}{y}\right) \cdot \frac{xy}{z}$ | | |
| 22) | $\frac{3x}{5(x-y)} * \frac{20(x-y)}{21y}$ | | |

$$42) \quad \frac{a^2 - b^2}{a - 2b} \cdot \frac{a^2 - 4b^2}{4a - 4b}$$

$$43) \quad \frac{2 - b}{a - 1} \cdot \frac{a^2 - 1}{4 - b^2}$$

$$44) \quad \frac{r^2 - s^2}{r - rs} : \frac{r - s}{r^2 + s^2}$$

$$45) \quad \frac{(r-s)^2}{r^2 - rs} : \frac{r - s}{r^2 + s^2}$$

$$46) \quad \frac{p^2 - 2p}{p^2 - 4} : \frac{p}{p + 2}$$

$$47) \quad \frac{16x^2 - 4}{16x - 8} : \frac{4x^2 - 1}{2x - 1}$$

$$48) \quad I_1R_1 + I_2R_3 + I_4R_5 + I_1R_2 + I_3R_3 + I_3R_4 + I_4R_6$$

$$49) \quad I_1R_1 - U + I_1R_2 + I_6R_2 - I_5R_3 + I_1R_3 + I_5R_3 - I_5R_4 + I_1R_4$$

50) $G = Z =$ Menge aller ganzen Zahlen. Bestimmen Sie die Lösungsmengen.

a) $7x - (2x - 9) + (3x + 8) - (5x + 6) = 5$

b) $21x - [9 - (5x - 6) + 8x] = 15 - [(4x - 7) - (6x - 5)]$

c) $12x - [14 - (9x - 11)] = 24x - [18 - (17x + 14)]$

d) $5(8x + 5) - 4(3x + 4) - 2(11x - 17) = 25 - 3(5x - 7) + 6(3x - 2)$

Lösungen: (bitte Kontrollieren)

1)

$$\begin{aligned}2d + (5d + 4e) - (10d - 8e - 38) - 6d - 17 &= \\2d + 5d + 4e - 10d + 8e + 38 - 6d - 17 &= \\-9d + 12e + 21 &= \end{aligned}$$

2)

$$\begin{aligned}7 - \{[(26x + 37y - 25z) + 19y - 16a] - 8x + 9z + 6\} &= \\7 - [(26x + 37y - 25z) + 19y - 16a] + 8x - 9z - 6 &= \\7 - (26x + 37y - 25z) - 19y + 16a + 8x - 9z - 6 &= \\7 - 26x - 37y + 25z - 19y + 16a + 8x - 9z - 6 &= \\1 - 18x - 56y + 16z + 16a &= \end{aligned}$$

3)

$$\begin{aligned}(3a + 4x) - \{6a - [5x - (9a - 8x)] + 13a\} &= \\3a + 4x - 6a + [5x - (9a - 8x)] - 13a &= \\3a + 4x - 6a + 5x - (9a - 8x) - 13a &= \\3a + 4x - 6a + 5x - 9a + 8x - 13a &= \\-25a + 17x &= \end{aligned}$$

4)

$$\begin{aligned}4x + 6y - \{6x - [7y - (5x + 3y) - (6y - 8x) - 3x] - 3x\} &= \\4x + 6y - 6x + [7y - (5x + 3y) - (6y - 8x) - 3x] + 3x &= \\4x + 6y - 6x + 7y - (5x + 3y) - (6y - 8x) - 3x + 3x &= \\4x + 6y - 6x + 7y - 5x - 3y - 6y + 8x &= \\x + 4y &= \end{aligned}$$

5)

$$\begin{aligned}4xy - 8xz - \{2yz - [3xy - (5xz + 6yz) + 7yz] - 2xy\} &= \\4xy - 8xz - 2yz + [3xy - (5xz + 6yz) + 7yz] + 2xy &= \\4xy - 8xz - 2yz + 3xy - (5xz + 6yz) + 7yz + 2xy &= \\4xy - 8xz - 2yz + 3xy - 5xz - 6yz + 7yz + 2xy &= \\4xy - 8xz - 2yz + 3xy - 5xz - 6yz + 7yz + 2xy &= \\9xy - 13xz - yz &= \end{aligned}$$

6)

$$\begin{aligned}\{[4c - (5cd + d)] - [7d - (c - 2cd)]\} - c - d &= \\4c - (5cd + d) - 7d + (c - 2cd) - c - d &= \\4c - 5cd - d - 7d + c - 2cd - c - d &= \\4c - 7cd - 9d &= \end{aligned}$$

7)

$$(-5i)(18a + 12b - 14c + 15x) = \\ -90ia - 60bi + 70ci - 75xi$$

8)

$$(y - 9) \cdot (x - 4) = \\ xy - 4y - 9x + 36$$

9)

$$(a + b)(4x - 5y) - (a - b)(5x + 3y) = \\ 4ax - 5ay + 4bx - 5by - (5ax + 3ay - 5bx - 3by) = \\ 4ax - 5ay + 4bx - 5by - 5ax - 3ay + 5bx + 3by = \\ -ax - 8ay + 9bx - 2by$$

10)

$$(3a - 5b)(6x - 7y + 9z) - (5x - 8y + 8z)(4a - 5b) = \\ 18ax - 21ay + 27az - 30bx + 35by - 45bz - (20ax - 25bx - 32ay + 40by + 32az - 40bz) \\ 18ax - 21ay + 27az - 30bx + 35by - 45bz - 20ax + 25bx + 32ay - 40by - 32az + 40bz \\ - 2ax + 11ay - 5az - 5bx - 5by - 5bz$$

11)

$$(4a + 3b - 5c)7x - [(5a - 4b + 6c)3x] = \\ 28ax + 21bx - 35cx - 3x(5a - 4b + 6c) = \\ 28ax + 21bx - 35cx - 15ax + 12bx - 18cx = \\ 13ax + 33bx - 53cx$$

12)

$$(4 + 2a - 3c) \cdot (12 - 2d - 5b) - [(12d - 6b) \cdot 9a] = \\ 48 - 8d - 20b + 24a - 4ad - 10ab - 36c + 6cd + 15bc - 9a(12d - 6b) = \\ 48 - 8d - 20b + 24a - 4ad - 10ab - 36c + 6cd + 15bc - 108ad + 54ab = \\ 48 - 8d - 20b + 24a - 112ad + 44ab - 36c + 6cd + 15bc$$

13)

$$(3a - 2b)(2c - 4d)(5x - 2y) = \\ (6ac - 12ad - 4bc + 8bd)(5x - 2y) = \\ 30acx - 60adx - 20bcx + 40bdx - 12acy + 24ady + 8bcy - 16bdy =$$

14)

$$21abx - 6by + 15bz \\ 21abx - 6by + 15bz = \\ 3b(7ax - 2y + 5z)$$

15)

$$25ab + 125ac + 75ax =$$

$$25a(b + 5c + 3x)$$

16)

$$axnd - axnc + abnd - abnc =$$

$$an(xd - xc + bd + bc)$$

17)

$$R_{20} + R_{20} \cdot \alpha \cdot \Delta\vartheta + R_{20} \cdot \beta \cdot (\Delta\vartheta)^2 =$$

$$R_{20}(1 + \alpha \cdot \Delta\vartheta + \beta \cdot (\Delta\vartheta)^2)$$

18)

$$\frac{a+b}{5x} + \frac{6a-5b}{5x} - \frac{2a+b}{5x}$$

$$\frac{a+b+6a-5b-(2a+b)}{5x} =$$

$$\frac{a+b+6a-5b-2a-b}{5x} =$$

$$\frac{5a-5b}{5x} = \frac{5(a-b)}{5x} = \frac{a-b}{x}$$

19)

$$\frac{a}{3x} - \frac{2a}{2x} + \frac{3a}{x} =$$

$$\frac{2a-6a+18a}{6x} =$$

$$\frac{14a}{6x} = \frac{7a}{3x}$$

20)

$$\frac{6b+2}{4y-8} + \frac{5b-3}{2y-4} - \frac{3b-1}{y-2} =$$

$$\frac{6b+2}{4y-8} + \frac{2(5b-3)}{2(2y-4)} - \frac{4(3b-1)}{4(y-2)} =$$

$$\frac{6b+2}{4y-8} + \frac{2(5b-3)}{4y-8} - \frac{4(3b-1)}{4y-8} =$$

$$\frac{6b+2+2(5b-3)-4(3b-1)}{4y-8} =$$

$$\frac{6b+2+10b-6-12b+4}{4y-8} =$$

$$\frac{4b}{4y-8} = \frac{4b}{4(y-2)} = \frac{b}{y-2}$$

21)

$$\left(\frac{a}{x} - \frac{b}{y}\right) \cdot \frac{xy}{z} =$$

$$\frac{axy}{xz} - \frac{bxy}{yz} =$$

$$\frac{ay}{z} - \frac{bx}{z} = \frac{ay - bx}{z}$$

22)

$$\frac{3x}{5(x-y)} \cdot \frac{20(x-y)}{21y} =$$

$$\frac{3x \cdot 20(x-y)}{5(x-y) \cdot 21y} = \frac{3x \cdot 4(x-y)}{(x-y) \cdot 21y} =$$

$$\frac{x \cdot 4(x-y)}{(x-y) \cdot 7y} = \frac{4x(x-y)}{7y(x-y)} =$$

$$\frac{4x}{7y}$$

23)

$$\frac{a}{y} : \frac{b}{y} = \frac{a}{y} \cdot \frac{y}{b} = \frac{a}{b}$$

24)

$$\frac{4ax}{a-x} : \frac{8ay}{a-x} =$$

$$\frac{4ax}{a-x} \cdot \frac{a-x}{8ay} =$$

$$\frac{4ax \cdot (a-x)}{(a-x) \cdot 8ay} =$$

$$\frac{4ax}{8ay} = \frac{x}{2y}$$

25)

$$\left(\frac{a}{1} - \frac{x}{y}\right) : \frac{a}{y} =$$

$$\left(\frac{a}{1} - \frac{x}{y}\right) \cdot \frac{y}{a} =$$

$$\frac{ay}{a} - \frac{xy}{ay} = y - \frac{x}{a}$$

26)

$$(x-3)^2 =$$

$$x^2 - 6x + 9$$

27)

$$(7n + 8m)^2 \\ 49n^2 + 112nm + 64m^2$$

28)

$$(3x^2 - 4x^2)^2 = (-x^2)^2 = x^4$$

29)

$$(y - 3)(y + 3) = \\ y^2 - 9$$

30)

$$(3n + m)^3 = (3n + m)^2 \cdot (3n + m) = \\ ((3n)^2 + 2 \cdot 3n \cdot m + m^2)(3n + m) = \\ (9n^2 + 6nm + m^2)(3n + m) = \\ 27n^3 + 18n^2m + 3nm^2 + 9n^2m + 6nm^2 + m^3 = \\ 27n^3 + 27n^2m + 9nm^2 + m^3$$

31)

$$(5m + 4)(5m - 4) \\ 25m^2 - 16$$

32)

$$(-x - y)^2 = (-x + (-y))^2 = \\ (-x)^2 + 2(-x)(-y) + (-y)^2 = \\ x^2 + 2xy + y^2$$

oder:

$$(-x - y)^2 = (-(x + y))^2 = (x + y)^2 = \\ x^2 + 2xy + y^2$$

33)

$$(5m + 7n)^2 - (3m - 4n)^2 = \\ (25m^2 + 70nm + 49n^2) - (9m^2 - 24nm + 16n^2) = \\ 25m^2 + 70nm + 49n^2 - 9m^2 + 24nm - 16n^2 = \\ 16m^2 + 94nm + 33n^2$$

34)

$$(3x - 7y)^2 - (7x - 3y)^2 - (7x - 3y)(3y + 7x) = \\ 9x^2 - 42xy + 49y^2 - (49x^2 - 42xy + 9y^2) - (49x^2 - 9y^2) = \\ 9x^2 - 42xy + 49y^2 - 49x^2 + 42xy - 9y^2 - 49x^2 + 9y^2 = \\ -99x^2 + 49y^2$$

35)

$$\begin{aligned} \frac{a^2+b^2}{a^2-b^2} \cdot \frac{a-b}{a+b} &= \\ \frac{a^2+b^2}{(a-b)(a+b)} \cdot \frac{a-b}{a+b} &= \\ \frac{(a^2+b^2)(a-b)}{(a-b)(a+b)(a+b)} &= \\ \frac{(a^2+b^2)}{(a+b)(a+b)} &= \frac{(a^2+b^2)}{(a+b)^2} \end{aligned}$$

36)

$$\begin{aligned} \frac{(a+b)^2}{a^2-b^2} \cdot \frac{a-b}{a+b} &= \frac{(a+b)^2 \cdot (a-b)}{a^2-b^2 \cdot (a+b)} = \\ \frac{(a+b)^2 \cdot (a-b)}{(a-b)(a+b)(a+b)} &= 1 \end{aligned}$$

37)

$$\begin{aligned} \frac{4c^2-1}{d^2-4} \cdot \frac{d+2}{2c-1} &= \frac{(2c)^2-1^2}{d^2-2^2} \cdot \frac{d+2}{2c-1} = \\ \frac{(2c-1)(2c+1)}{(d-2)(d+2)} \cdot \frac{d+2}{2c-1} &= \frac{(2c-1)(2c+1) \cdot (d+2)}{(d-2)(d+2) \cdot (2c-1)} = \\ \frac{2c+1}{d-2} \end{aligned}$$

38)

$$\begin{aligned} \frac{x^2+1+2x}{x^2+1} \cdot \frac{(x-1)^2}{(x+1)^2} &= \frac{(x+1)^2}{x^2+1} \cdot \frac{(x-1)^2}{(x+1)^2} = \\ \frac{(x-1)^2}{x^2+1} \end{aligned}$$

39)

$$\begin{aligned}\frac{(2x-6y)^2}{(x-y)^2} \cdot \frac{x-3y}{x^2-y^2} &= \\ \frac{(2x-6y)^2 \cdot (x^2-y^2)}{(x-y)^2 \cdot (x-3y)} &= \\ \frac{(2(x-3y))^2 \cdot (x-y)(x+y)}{(x-y)^2 \cdot (x-3y)} &= \\ \frac{4(x-3y)^2 \cdot (x-y)(x+y)}{(x-y)^2 \cdot (x-3y)} &= \\ \frac{4(x-3y)(x+y)}{x-y} &= \end{aligned}$$

40)

$$\begin{aligned}\frac{x^2-x^2y^2}{x^2+x^2y} \cdot \frac{x+xy}{x^2+x^2y} &= \\ \frac{x^2-x^2y^2}{x^2+x^2y} \cdot \frac{x^2+x^2y}{x+xy} &= \\ \frac{(x^2-x^2y^2) \cdot (x^2+x^2y)}{(x^2+x^2y)(x+xy)} &= \\ \frac{(x^2-x^2y^2)}{(x+xy)} = \frac{x^2(1-y^2)}{x(1+y)} &= \\ \frac{x^2(1-y)(1+y)}{x(1+y)} = x(1-y) &= \end{aligned}$$

41)

$$\begin{aligned}\frac{p^2-4p}{p^2-4} \cdot \frac{p+2}{p} &= \\ \frac{p^2-4p}{p^2-2^2} \cdot \frac{p+2}{p} &= \\ \frac{p^2-4p}{(p-2)(p+2)} \cdot \frac{p+2}{p} &= \\ \frac{p(p-4) \cdot (p+2)}{(p-2)(p+2) \cdot p} = \frac{p-4}{p-2} &= \end{aligned}$$

42)

$$\begin{aligned}\frac{a^2 - b^2}{a - 2b} \cdot \frac{a^2 - 4b^2}{4a - 4b} &= \\ \frac{(a - b)(a + b)}{a - 2b} \cdot \frac{a^2 - (2b)^2}{4(a - b)} &= \\ \frac{(a - b)(a + b)}{a - 2b} \cdot \frac{(a - 2b)(a + 2b)}{4(a - b)} &= \\ \frac{(a - b)(a + b) \cdot (a - 2b)(a + 2b)}{(a - 2b) \cdot 4(a - b)} &= \\ \frac{(a + b)(a + 2b)}{4} &= \end{aligned}$$

43)

$$\begin{aligned}\frac{2 - b}{a - 1} \cdot \frac{a^2 - 1}{4 - b^2} &= \\ \frac{2 - b}{a - 1} \cdot \frac{a^2 - 1^2}{2^2 - b^2} &= \\ \frac{2 - b}{a - 1} \cdot \frac{(a - 1)(a + 1)}{(2 - b)(2 + b)} &= \\ \frac{(2 - b) \cdot (a - 1)(a + 1)}{(a - 1) \cdot (2 - b)(2 + b)} &= \frac{a + 1}{2 + b} \end{aligned}$$

44)

$$\begin{aligned}\frac{r^2 - s^2}{r - rs} \div \frac{r - s}{r^2 + s^2} &= \\ \frac{r^2 - s^2}{r - rs} \cdot \frac{r^2 + s^2}{r - s} &= \\ \frac{(r - s)(r + s)}{r(1 - s)} \cdot \frac{r^2 + s^2}{r - s} &= \\ \frac{(r - s)(r + s) \cdot (r^2 + s^2)}{r(1 - s) \cdot (r - s)} &= \\ \frac{(r + s) \cdot (r^2 + s^2)}{r(1 - s)} &= \end{aligned}$$

45)

$$\begin{aligned}\frac{(r-s)^2}{r^2-rs} \cdot \frac{r-s}{r^2+s^2} &= \\ \frac{(r-s)^2}{r^2-rs} \cdot \frac{r^2+s^2}{r-s} &= \\ \frac{(r-s)^2}{r(r-s)} \cdot \frac{r^2+s^2}{r-s} &= \\ \frac{(r-s)^2 \cdot (r^2+s^2)}{r(r-s) \cdot (r-s)} &= \\ \frac{r^2+s^2}{r} &= \end{aligned}$$

46)

$$\begin{aligned}\frac{p^2-2p}{p^2-4} \cdot \frac{p}{p+2} &= \\ \frac{p(p-2)}{p^2-2^2} \cdot \frac{p+2}{p} &= \\ \frac{p(p-2)}{(p-2)(p+2)} \cdot \frac{p+2}{p} &= \\ \frac{p(p-2) \cdot (p+2)}{(p-2)(p+2) \cdot p} &= 1 \end{aligned}$$

47)

$$\begin{aligned}\frac{16x^2-4}{16x-8} \cdot \frac{4x^2-1}{2x-1} &= \\ \frac{16x^2-4}{16x-8} \cdot \frac{2x-1}{4x^2-1} &= \\ \frac{(4x)^2-2^2}{8(2x-1)} \cdot \frac{2x-1}{(2x)^2-1^2} &= \\ \frac{(4x-2)(4x+2)}{8(2x-1)} \cdot \frac{2x-1}{(2x-1)(2x+1)} &= \\ \frac{(4x-2)(4x+2) \cdot (2x-1)}{8(2x-1) \cdot (2x-1)(2x+1)} &= \\ \frac{2(2x-1) \cdot 2(2x-1) \cdot (2x-1)}{8(2x-1) \cdot (2x-1)(2x+1)} &= \\ \frac{2 \cdot 2 \cdot (2x-1)}{8(2x+1)} = \frac{2x-1}{2(2x+1)} &= \end{aligned}$$

48)

$$\begin{aligned}I_1R_1 + I_2R_3 + I_4R_5 + I_1R_2 + I_3R_3 + I_3R_4 + I_4R_6 &= \\ I_1(R_1 + R_2) + R_3(I_2 + I_3) + I_4(R_5 + R_6) + I_3 + R_4 &= \end{aligned}$$

49)

$$I_1 R_1 - U + I_1 R_2 + I_6 R_2 - I_5 R_3 + I_1 R_3 + I_5 R_3 - I_5 R_4 + I_1 R_4 =$$
$$I_1 (R_1 + R_4) - I_5 (R_4 + R_3) + R_3 (I_5 I_1) + R_2 (I_1 + I_6) - U$$

50)

a)

$$7x - (2x - 9) + (3x + 8) - (5x + 6) = 5$$

$$7x - 2x + 9 + 3x + 8 - 5x - 6 = 5$$

$$3x + 11 = 5$$

$$3x = -6$$

$$x = -2$$

$$L = \{-2\}$$

b)

$$21x - [9 - (5x - 6) + 8x] = 15 - [(4x - 7) - (6x - 5)]$$

$$21x - 9 + 5x - 6 - 8x = 15 - 4x + 7 + 6x - 5$$

$$18x - 15 = 17 + 2x$$

$$16x = 32$$

$$x = 2$$

c)

$$12x - [14 - (9x - 11)] = 24x - [18 - (17x + 14)]$$

$$12x - 14 + (9x - 11) = 24x - 18 + (17x + 14)$$

$$12x - 14 + 9x - 11 = 24x - 18 + 17x + 14$$

$$21x - 25 = 41x - 4$$

$$20x = -21$$

$$x = -\frac{21}{20}$$

$$L = \{ \}$$

d)

$$5(8x + 5) - 4(3x + 4) - 2(11x - 17) = 25 - 3(5x + 7) + 6(3x + 2)$$

$$40x + 25 - 12x - 16 - 22x + 34 = 25 - 15x - 21 + 18x + 12$$

$$6x + 43 = 16 + 3x$$

$$3x = -27$$

$$x = -9$$

$$L = \{-9\}$$

4 ÜBUNGSAUFGABEN MESK 2BK11

Die Grundmenge ist $G = \mathbb{R}$. Bestimmen Sie die Definitionsmenge D und die Lösungsmenge L der Gleichungen
 Siehe Bemerkungen Rückseite

- 1) $\frac{x}{3} + \frac{x}{4} = 7$
- 2) $\frac{x}{2} - \frac{x}{5} = 6$
- 3) $\frac{7x}{10} - \frac{2}{5} = \frac{x}{2}$
- 4) $\frac{3}{4} - \frac{x}{2} = \frac{5x}{6} + \frac{7}{12}$
- 5) $\frac{3x}{5} - \frac{x}{6} - \frac{5x}{12} - \frac{x}{15} = x - 9$
- 6) $\frac{1}{2}x - \frac{2}{3}x + \frac{3}{4}x - \frac{5}{6}x + \frac{7}{12}x = \frac{11}{12}$
- 7) $\frac{x+4}{14} + \frac{x-4}{6} = 2$
- 8) $\frac{x-3}{18} - \frac{2x+7}{8} = \frac{2}{9}$
- 9) $\frac{x+1}{3} - \frac{3x-1}{5} = x - 2$
- 10) $\frac{3x-5}{4} + \frac{2x-3}{6} - \frac{x+6}{3} = \frac{x-3}{2}$
- 11) $2x - \frac{1}{2} - \frac{3}{4}x + \frac{2}{3} = \frac{1}{6} + \frac{5}{4}x$
- 12) $\frac{x-3}{6} - \frac{2x-5}{3} = \frac{4-x}{2} - \frac{3}{2}$
- 13) $\frac{5+3x}{18} - \frac{x}{2} = \frac{2-3x}{9}$
- 14) $\frac{4}{x} = 5$
- 15) $\frac{3}{2x} = 4$
- 16) $\frac{9}{2x} + 5 = \frac{17}{x}$
- 17) $\frac{1}{2} + \frac{7}{x} + \frac{1}{4} = \frac{12}{x} + \frac{3}{4}$
- 18) $\frac{1}{x} + \frac{1}{3x} = \frac{8}{9}$
- 19) $\frac{3}{4x} + \frac{3}{6x} = \frac{7}{8}$
- 20) $\frac{2}{x} = 1 + \frac{2}{x}$
- 21) $-\frac{1}{3} + \frac{3}{2x} + \frac{5}{6} - \frac{4}{3x} - \frac{7}{2} = \frac{11}{6x}$
- 22) $\frac{9}{10x} - \frac{1}{5} + \frac{3}{5x} + \frac{13}{15} - \frac{3}{2x} + \frac{1}{3} = 1$
- 23) $\frac{4}{x+5} = 1$
- 24) $\frac{7}{x-3} = 2$
- 25) $\frac{3}{x+2} + \frac{5}{2} = 0$
- 26) $\frac{2}{x+1} = \frac{3}{x-1}$
- 27) $\frac{1}{x-1} = \frac{-1}{1-x}$
- 28) $\frac{-3}{2-x} = \frac{3}{x-2}$
- 29) $\frac{1}{x-3} - 2 = \frac{1}{2(x-3)}$
- 30) $\frac{15}{2x+5} - \frac{15}{4x+10} = \frac{19}{9x-7}$
- 31) $\frac{5x}{2x-2} - \frac{x}{3x-3} = 2$
- 32) $\frac{2-x}{3-x} - 1 = \frac{4-x}{x-3}$
- 33) $\frac{1}{\frac{3}{x}+1} + \frac{2}{x+3} = 1$
- 34) $\frac{x+6}{x} = \frac{x+4}{x+1}$
- 35) $\frac{3x+2}{3x-1} - \frac{6x}{6x-1} = 0$
- 36) $\frac{1}{x} + \frac{x}{x-1} = 1$
- 37) $\frac{3x}{x+1} + \frac{5}{x} = 3$
- 38) $\frac{3x+1}{4x-10} + \frac{5x-1}{6x-15} - \frac{7x+5}{10x-25} = \frac{8}{5}$
- 39) $\frac{3}{x} + \frac{5}{x^2} - \frac{3}{x-1} = \frac{2}{(x-1)^2}$
- 40) $\frac{3+7x}{1+x} - \frac{4-9x}{1-x} + 6 = \frac{15-4x^2}{1-x^2}$
- 41) $\frac{3}{x+4} - \frac{2}{x-4} = \frac{5x-20}{x^2-16}$
- 42) $\frac{3}{2} - \frac{8x^2-13x+21}{4x^2-20x+25} + \frac{x+4}{2x-5} = 0$

$$43) \frac{7(x-5)^2}{6x^2-6} = \frac{5x-1}{3x+3} - \frac{3x-2}{6x-6}$$

$$44) \frac{5x^2+32x+3}{x^2+4x+3} - 2 = \frac{3x+9}{x-1}$$

$$45) \frac{2}{x-1} + \frac{3}{x-2} = \frac{5}{x-3}$$

$$46) \frac{4}{x+2} - \frac{1}{x-1} = \frac{3}{x+1}$$

PROBEmachen!!!

Bemerkungen:

Machen Sie Proben (in der **Ausgangsgleichung**) wie folgt:

1)

a) Fall: Lösungsmenge ist die leere Menge

Man müsste für alle reelle Zahlen Probe machen und dann müsste **IMMER** eine falsche Aussage entstehen (links und rechts des Gleichheitszeichens eine unterschiedliche Zahl sein). Testen Sie dies für 3 Zahlen!

b) Fall: Lösungsmenge ist die Menge der reellen Zahlen

Man müsste für alle reelle Zahlen Probe machen und dann müsste **IMMER** eine wahre Aussage entstehen (links und rechts des Gleichheitszeichens die gleiche Zahl sein). Testen Sie dies für 3 Zahlen!

c) Fall: Lösungsmenge besteht aus endlich vielen Zahlen

Machen Sie die Probe für alle diese Zahlen.

2)

Falls bei der Probe festgestellt wird, dass ein Fehler gemacht wurde, muss bei jeder Umformung eine (oder mehrere) Proben gemacht werden, bis man festgestellt hat, wo sich der Fehler befindet.

Lösungen:

1)

$$\frac{x}{3} + \frac{x}{4} = 7 \mid \cdot 12$$

$$D = R$$

$$4x + 3x = 84$$

$$7x = 84$$

$$x = 12$$

$$L = \{12\}$$

2)

$$\frac{x}{2} - \frac{x}{5} = 6 \mid \cdot 10$$

$$D = R$$

$$5x - 2x = 60$$

$$3x = 60$$

$$x = 20$$

$$L = \{20\}$$

3)

$$\frac{7x}{10} - \frac{2}{5} = \frac{x}{2} \mid \cdot 10$$

$$D = R$$

$$7x - 4 = 5x$$

$$1 \quad 7x - 5x = 4$$

$$2x = 4$$

$$x = 2$$

$$L = \{2\}$$

4)

$$\frac{3}{4} - \frac{x}{2} = \frac{5x}{6} + \frac{7}{12} \mid \cdot 12$$

$$D = R$$

$$9 - 6x = 10x + 7$$

$$16x = 2$$

$$x = \frac{1}{8}$$

$$L = \left\{ \frac{1}{8} \right\}$$

5)

$$\frac{3x}{5} - \frac{x}{6} - \frac{5x}{12} - \frac{x}{15} = x - 9 \mid \cdot 60$$

$$D = R$$

$$(12 \cdot 3x) - (10 \cdot x) - (5 \cdot 5x) - (4 \cdot x) = 60 \cdot (x - 9)$$

$$36x - 10x - 25x - 4x = 60x - 540$$

$$63x = 540$$

$$x = \frac{60}{7}$$

$$L = \left\{ \frac{60}{7} \right\}$$

6)

$$\frac{1}{2}x - \frac{2}{3}x + \frac{3}{4}x - \frac{5}{6}x + \frac{7}{12}x = \frac{11}{12} \mid \cdot 12$$

$$D = R$$

$$6x - 8x + 9x - 10x + 7x = 11$$

$$4x = 11$$

$$x = \frac{11}{4}$$

$$L = \left\{ \frac{11}{4} \right\}$$

7)

$$\frac{x+4}{14} + \frac{x-4}{6} = 2 \mid \cdot 42$$

$$D = R$$

$$3 \cdot (x+4) + 7(x-4) = 84$$

$$3x + 12 + 7x - 28 = 84$$

$$10x = 100$$

$$x = 10$$

$$L = \{10\}$$

8)

$$\frac{x-3}{18} - \frac{2x+7}{8} = \frac{2}{9} \quad | \cdot 72$$

$$D = R$$

$$4 \cdot (x-3) - 9 \cdot (2x+7) = 16$$

$$4x - 12 - 18x - 63 = 16$$

$$-14x = 91 \quad | \cdot (-1)$$

$$14x = -91$$

$$x = -\frac{13}{2}$$

$$L = \left\{-\frac{13}{2}\right\}$$

10)

$$\frac{3x-5}{4} + \frac{2x-3}{6} - \frac{x+6}{3} = \frac{x-3}{2} \quad | \cdot 12$$

$$D = R$$

$$3 \cdot (3x-5) + 2 \cdot (2x-3) - 4 \cdot (x+6) = 6 \cdot (x-3)$$

$$9x - 15 + 4x - 6 - 4x - 24 = 6x - 18$$

$$3x = 27$$

$$x = 9$$

$$L = \{9\}$$

12)

$$\frac{x-3}{6} - \frac{2x-5}{3} = \frac{4-x}{2} - \frac{3}{2} \quad | \cdot 6$$

$$D = R$$

$$x - 3 - 4x + 10 = 12 - 3x - 9$$

$$0 \cdot x = -4$$

$$L = \{\}$$

9)

$$\frac{x+1}{3} - \frac{3x-1}{5} = x-2 \quad | \cdot 15$$

$$D = R$$

$$5 \cdot (x+1) - 3 \cdot (3x-1) = 15 \cdot (x-2)$$

$$5x + 5 - 9x + 3 = 15x - 30$$

$$19x = 38$$

$$x = 2$$

$$L = \{2\}$$

11)

$$2x - \frac{1}{2} - \frac{3}{4}x + \frac{2}{3} = \frac{1}{6} + \frac{5}{4}x \quad | \cdot 12$$

$$D = R$$

$$24x - 6 - 9x + 8 = 2 + 15x$$

$$0 \cdot x = 0$$

$$L = D = R$$

13)

$$\frac{5+3x}{18} - \frac{x}{2} = \frac{2-3x}{9} \quad | \cdot 18$$

$$D = R$$

$$5 + 3x - 9x = 2 \cdot (2 - 3x)$$

$$5 - 6x = 4 - 6x$$

$$0 \cdot x = 1$$

$$L = \{\}$$

14)

$$\frac{4}{x} = 5 \mid \cdot x$$

$$D = \mathbb{R} \setminus \{0\}$$

$$4 = 5x$$

$$x = \frac{4}{5}$$

$$L = \left\{ \frac{4}{5} \right\}$$

15)

$$\frac{3}{2x} = 4 \mid \cdot 2x$$

$$D = \mathbb{R} \setminus \{0\}$$

$$3 = 8x$$

$$x = \frac{3}{8}$$

$$L = \left\{ \frac{3}{8} \right\}$$

16)

$$\frac{9}{2x} + 5 = \frac{17}{x} \mid \cdot 2x$$

$$D = \mathbb{R} \setminus \{0\}$$

$$9 + 10x = 34$$

$$10x = 25$$

$$x = \frac{5}{2}$$

$$L = \left\{ \frac{5}{2} \right\}$$

17)

$$\frac{1}{2} + \frac{7}{x} + \frac{1}{4} = \frac{12}{x} + \frac{3}{4} \mid \cdot 4x$$

$$D = \mathbb{R} \setminus \{0\}$$

$$2x + 28 + x = 48 + 3x$$

$$0x = 20$$

$$L = \{ \}$$

18)

$$\frac{1}{x} + \frac{1}{3x} = \frac{8}{9} \mid \cdot 9x$$

$$D = \mathbb{R}$$

$$9 + 3 = 8x$$

$$8x = 12$$

$$x = \frac{3}{2}$$

$$L = \left\{ \frac{3}{2} \right\}$$

19)

$$\frac{3}{4x} + \frac{3}{6x} = \frac{7}{8} \mid \cdot 24x$$

$$D = \mathbb{R} \setminus \{0\}$$

$$18 + 12 = 21x$$

$$21x = 30$$

$$x = \frac{10}{7}$$

$$L = \left\{ \frac{10}{7} \right\}$$

20)

$$\frac{2}{x} = 1 + \frac{2}{x} \mid \cdot x$$

$$D = \mathbb{R} \setminus \{0\}$$

$$2 = x + 2$$

$$x = 0$$

$$L = \{ \}$$

21)

$$-\frac{1}{3} + \frac{3}{2x} + \frac{5}{6} - \frac{4}{3x} - \frac{7}{2} = \frac{11}{6x} \mid \cdot 6x$$

$$D = \mathbb{R} \setminus \{0\}$$

$$-2x + 9 + 5x - 8 - 21x = 11$$

$$-18x + 1 = 11$$

$$-18x = 10$$

$$x = -\frac{5}{9}$$

$$L = \left\{ -\frac{5}{9} \right\}$$

22)

$$\frac{9}{10x} - \frac{1}{5} + \frac{3}{5x} + \frac{13}{15} - \frac{3}{2x} + \frac{1}{3} = 1 \mid \cdot 30x$$

$$D = \mathbb{R} \setminus \{0\}$$

$$27 - 6x + 18 + 26x - 45 + 10x = 30x$$

$$30x = 30x$$

$$x = x$$

$$L = D$$

23)

$$\frac{4}{x+5} = 1 \mid \cdot x+5$$

$$D = \mathbb{R} \setminus \{-5\}$$

$$4 = x+5$$

$$x = -1$$

$$L = \{-1\}$$

24)

$$\frac{7}{x-3} = 2 \mid \cdot x-3$$

$$D = \mathbb{R} \setminus \{3\}$$

$$7 = 2x-6$$

$$2x = 13$$

$$x = \frac{13}{2}$$

$$L = \left\{\frac{13}{2}\right\}$$

25)

$$\frac{3}{x+2} + \frac{5}{2} = 0 \mid \cdot (x+2) \cdot 2$$

$$D = \mathbb{R} \setminus \{-2\}$$

$$6 + 5x + 10 = 0$$

$$5x = -16$$

$$x = -\frac{16}{5}$$

$$L = \left\{-\frac{16}{5}\right\}$$

26)

$$\frac{2}{x+1} = \frac{3}{x-1} \mid \cdot (x+1)(x-1)$$

$$D = \mathbb{R} \setminus \{1; -1\}$$

$$2x-2 = 3x+3$$

$$x = -5$$

$$L = \{-5\}$$

27)

$$\frac{1}{x-1} = \frac{-1}{1-x} \mid \cdot (1-x)(x-1)$$

$$D = \mathbb{R} \setminus \{1\}$$

$$1-x = -x+1$$

$$1-x = 1-x$$

$$L = D$$

28)

$$\frac{-3}{2-x} = \frac{3}{x-2} \mid \cdot (2-x)(x-2)$$

$$D = \mathbb{R} \setminus \{2\}$$

$$-3x+6 = 6-3x$$

$$6-3x = 6-3x$$

$$L = D$$

29)

$$\frac{1}{x-3} - 2 = \frac{1}{2(x-3)} \mid \cdot 2(x-3)$$

$$D = \mathbb{R} \setminus \{3\}$$

$$2-4(x-3) = 1$$

$$2-4x+12 = 1$$

$$4x = 13$$

$$x = \frac{13}{4}$$

$$L = \left\{\frac{13}{4}\right\}$$

30)

$$\frac{15}{2x+5} - \frac{15}{4x+10} = \frac{19}{9x-7}$$

$$D = \mathbb{R} \setminus \left\{ -\frac{5}{2}; \frac{7}{9} \right\}$$

HN bestimmen:

$2x+5$	$2x+5$		
$4x+10$	$2x+5$		2
$9x-7$		$9x-7$	
HN	$2x+5$	$9x-7$	2

$$\frac{15}{2x+5} - \frac{15}{2(2x+5)} = \frac{19}{9x-7} \quad | \cdot 2(2x+5)(9x-7)$$

$$15 \cdot 2(9x-7) - 15 \cdot (9x-7) = 19 \cdot 2(2x+5)$$

$$30(9x-7) - 15(9x-7) = 38(2x+5)$$

$$15(9x-7) = 38(2x+5)$$

$$135x - 105 = 76x + 190$$

$$59x = 295$$

$$x = 5$$

$$L = \{5\}$$

32)

$$\frac{2-x}{3-x} - 1 = \frac{4-x}{x-3}$$

$$D = \mathbb{R} \setminus \{3\}$$

$$\frac{2-x}{3-x} - 1 = \frac{4-x}{-(x-3)}$$

$$\frac{2-x}{3-x} - 1 = \frac{-(4-x)}{3-x} \quad | \cdot (3-x)$$

$$2-x-1(3-x) = -(4-x)$$

$$2-x-3+x = -4+x$$

$$-1 = x-4$$

$$x = 3$$

$$L = \{ \}$$

31)

$$\frac{5x}{2x-2} - \frac{x}{3x-3} = 2$$

$$D = \mathbb{R} \setminus \{1\}$$

HN bestimmen:

$2x-2$	$x-1$	2	
$3x-3$	$x-1$		3
HN	$x-1$	2	3

$$\frac{5x}{2(x-1)} - \frac{x}{3(x-1)} = 2 \quad | \cdot 6(x-1)$$

$$3 \cdot 5x - 2x = 2 \cdot 6(x-1)$$

$$15x - 2x = 12(x-1)$$

$$13x = 12x - 12$$

$$x = -12$$

$$L = \{-12\}$$

33)

$$\frac{1}{\frac{3}{x}+1} + \frac{2}{x+3} = 1$$

$$D = \mathbb{R} \setminus \{0; -3\}$$

$$\frac{1}{\frac{3+x}{x}} + \frac{2}{x+3} = 1$$

$$\frac{x}{3+x} + \frac{2}{x+3} = 1 \quad | \cdot (x+3)$$

$$x+2 = x+3$$

$$x-x = 1$$

$$0 = 1$$

$$L = \{ \}$$

34)

$$\frac{x+6}{x} = \frac{x+4}{x+1} \quad | \cdot x(x+1)$$

$$D = \mathbb{R} \setminus \{0; -1\}$$

$$(x+6)(x+1) = x(x+4)$$

$$x^2 + 6x + x + 6 = x^2 + 4x$$

$$7x + 6 = 4x$$

$$3x = -6$$

$$x = -2$$

$$L = \{-2\}$$

36)

$$\frac{1}{x} + \frac{x}{x-1} = 1 \quad | \cdot x(x-1)$$

$$D = \mathbb{R} \setminus \{0; 1\}$$

$$1(x-1) + x \cdot x = x(x-1)$$

$$x-1 + x^2 = x^2 - x$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$$L = \left\{ \frac{1}{2} \right\}$$

38)

$$\frac{3x+1}{4x-10} + \frac{5x-1}{6x-15} - \frac{7x+5}{10x-25} = \frac{8}{5}$$

$$D = \mathbb{R} \setminus \left\{ \frac{5}{2} \right\}$$

HN bestimmen:

4x-10	2x-5	2		
6x-15	2x-5		3	
10x-25	2x-5			5
HN	2x+5	2	3	5

$$\frac{3x+1}{2(2x-5)} + \frac{5x-1}{3(2x-5)} - \frac{7x+5}{5(2x-5)} = \frac{8}{5} \quad | \cdot 30(2x-5)$$

$$(3x+1) \cdot 15 + (5x-1) \cdot 10 - (7x+5) \cdot 6 = 8 \cdot 6 \cdot (2x-5)$$

$$45x + 15 + 50x - 10 - 42x - 30 = 48(2x-5)$$

$$53x - 25 = 96x - 240$$

$$215 = 43x$$

$$x = 5$$

$$L = \{5\}$$

35)

$$\frac{3x+2}{3x-1} - \frac{6x}{6x-1} = 0 \quad | \cdot (3x-1)(6x-1)$$

$$D = \mathbb{R} \setminus \left\{ \frac{1}{3}; \frac{1}{6} \right\}$$

$$(3x+2)(6x-1) - 6x(3x-1) = 0$$

$$18x^2 + 12x - 3x - x - 18x^2 + 6x = 0$$

$$15x = 2$$

$$x = \frac{2}{15}$$

$$L = \left\{ \frac{2}{15} \right\}$$

37)

$$\frac{3x}{x+1} + \frac{5}{x} = 3 \quad | \cdot x(x+1)$$

$$D = \mathbb{R} \setminus \{-1; 0\}$$

$$3x \cdot x + 5 \cdot (x+1) = 3x(x+1)$$

$$3x^2 + 5x + 5 = 3x^2 + 3x$$

$$2x = -5$$

$$x = -\frac{5}{2}$$

$$L = \left\{ -\frac{5}{2} \right\}$$

39)

$$\frac{3}{x} + \frac{5}{x^2} - \frac{3}{x-1} = \frac{2}{(x-1)^2} \quad | \cdot (x-1)^2 \cdot x^2$$

$$D = \mathbb{R} \setminus \{0; 1\}$$

$$3x(x-1)^2 + 5(x-1)^2 - 3x^2(x-1) = 2x^2$$

$$3x(x^2 - 2x + 1) + 5(x^2 - 2x + 1) - 3x^3 + 3x^2 = 2x^2$$

$$3x^3 - 6x^2 + 3x + 5x^2 - 10x + 5 - 3x^3 + 3x^2 = 2x^2$$

$$2x^2 - 7x + 5 = 2x^2$$

$$5 = 7x \quad | : 7$$

$$x = \frac{5}{7}$$

$$L = \left\{ \frac{5}{7} \right\}$$

HN bestimmen:

x	x			
x ²	x	x		
x-1			x-1	
(x-1) ²			x-1	x-1
HN	x	x	x-1	x-1

40)

$$\frac{3+7x}{1+x} - \frac{4-9x}{1-x} + 6 = \frac{15-4x^2}{1-x^2} \quad | \cdot (1+x)(1-x)$$

$$D = \mathbb{R} \setminus \{1; -1\}$$

$$(3+7x)(1-x) - (4-9x)(1+x) + 6(1+x)(1-x) = 15-4x^2$$

$$3-3x+7x-7x^2 - (4+4x-9x-9x^2) + 6-6x^2 = 15-4x^2$$

$$3-3x+7x-7x^2 - 4-4x+9x+9x^2 + 6-6x^2 = 15-4x^2$$

$$5+9x-4x^2 = 15-4x^2 \quad | +4x^2 \quad | -5$$

$$9x = 10 \quad | : 9$$

$$x = \frac{10}{9}$$

$$L = \left\{ \frac{10}{9} \right\}$$

HN bestimmen:

1+x	1+x	
1-x		1-x
1-x ²	1+x	1-x
HN	1+x	1-x

41)

$$\frac{3}{x+4} - \frac{2}{x-4} = \frac{5x-20}{x^2-16} \quad | \cdot (x^2-16)$$

$$D = \mathbb{R} \setminus \{4; -4\}$$

$$3(x-4) - 2(x+4) = 5x-20$$

$$3x-12-2x-8 = 5x-20$$

$$x-20 = 5x-20 \quad | \cdot +20 \quad | \cdot -5x$$

$$-4x = 0$$

$$L = \{0\}$$

HN bestimmen:

x+4	x+4	
x-4		x-4
x ² -16	x+4	x-4
HN	x+4	x-4

42)

$$\frac{3}{2} - \frac{8x^2 - 13x + 21}{4x^2 - 20x + 25} + \frac{x+4}{2x-5} = 0 \quad | \cdot 2(2x-5)^2$$

$$D = R \setminus \left\{ \frac{5}{2} \right\}$$

$$3(2x-5)^2 - 2(8x^2 - 13x + 21) + 2(x+4)(2x-5) = 0$$

$$3(4x^2 - 20x + 25) - 16x^2 + 26x - 42 + (2x+8)(2x-5) = 0$$

$$12x^2 - 60x + 75 - 16x^2 + 26x - 42 + 4x^2 - 10x + 16x - 40 = 0$$

$$-28x - 7 = 0 \quad | +7$$

$$-28x = 7 \quad | :(-28)$$

$$x = -\frac{1}{4}$$

$$L = \left\{ -\frac{1}{4} \right\}$$

Bem:

$$4x^2 - 20x + 25 =$$

$$(2x)^2 - 2 \cdot 2x \cdot 5 + 5^2 =$$

$$(2x-5)^2$$

HN bestimmen:

$4x^2 - 20x + 25$	$2x-5$	$2x-5$
$2x - 5$	$2x-5$	
HN	$2x-5$	$2x-5$

43)

$$\frac{7(x-5)^2}{6x^2-6} = \frac{5x-1}{3x+3} - \frac{3x-2}{6x-6} \quad | \cdot 6(x-1)(x+1)$$

$$D = R \setminus \{1; -1\}$$

$$7(x-5)^2 = 2(5x-1)(x-1) - (3x-2)(x+1)$$

$$7(x^2 - 10x + 25) = 2(5x^2 - 5x - x + 1) - (3x^2 + 3x - 2x - 2)$$

$$7x^2 - 70x + 175 = 10x^2 - 10x - 2x + 2 - 3x^2 - 3x + 2x + 2$$

$$7x^2 - 70x + 175 = 7x^2 - 13x + 4 \quad | -7x^2 \quad | +70x \quad | -4$$

$$171 = 57x \quad | :57$$

$$x = 3$$

$$L = \{3\}$$

Bem:

$$6x^2 - 6 = 6(x^2-1) =$$

$$6(x-1)(x+1)$$

HN bestimmen:

$6x^2 - 6$	$x-1$	$x+1$	2	3
$3x+3$		$x+1$		3
$6x-6$	$x-1$		2	3
HN	$x-1$	$x+1$	2	3

Lösung mit ungeschickt gewähltem Hauptnenner:

$$\frac{7(x-5)^2}{6x^2-6} = \frac{5x-1}{3x+3} - \frac{3x-2}{6x-6} \quad | \cdot (6x^2-6)(3x+3)(6x-6)$$

$$7(x-5)^2(3x+3)(6x-6) = (5x-1)(6x^2-6) - (3x-2)(6x^2-6)(3x+3)$$

$$7(x^2-10x+25)(3x+3)(6x-6) = (5x-1)(6x^2-6) - (3x-2)(6x^2-6)(3x+3)$$

$$7x^2-70x+175(3x+3)(6x-6) = (5x-1)(6x^2-6) - (3x-2)(6x^2-6)(3x+3)$$

$$7x^2-70x+175(3x+3)(6x-6) = (30x^2-6x^2+6-30x)(6x-6) - (18x^3-12x^2-18x+12)(3x+3)$$

$$(21x^3-210x^2+525x+21x^2-210x+525)(6x-6) =$$

$$(30x^3-6x^2-30x+6)(6x-6) - (18x^3-12x^2-18x+12)(3x+3)$$

$$126x^4 - 1134x^3 + 1890x^2 + 3150x - 126x^3 + 1134x^2 - 1890x - 3150 =$$

$$(18x^4 - 36x^3 - 180x^2 + 36x - 180x^3 + 36x^2 + 180x - 36) - (54x^4 - 36x^3 - 54x^2 + 36x + 54x^3 - 36x^2 - 54x + 36)$$

$$126x^4 - 1260x^3 + 1890x^2 + 3150x - 126x^3 + 1134x^2 - 1890x - 3150 =$$

$$180x^4 - 216x^3 - 144x^2 + 216x - 36 - (54x^4 + 18x^3 - 90x^2 - 18x - 36)$$

$$126x^4 - 1260x^3 + 3024x^2 + 1260x - 3150 =$$

$$180x^4 - 216x^3 - 144x^2 + 216x - 36 - 54x^4 - 18x^3 + 90x^2 + 18x + 36$$

$$\begin{aligned}
126x^4 - 1260x^3 + 3024x^2 + 1260x - 3150 &= \\
126x^4 - 234x^3 - 54x^2 + 234x - 72 & \quad | +234x^3 ; +54x^2 ; -234x ; +72 ; -126x^4 \\
-1026x^3 + 3078x^2 + 1026x - 3078 &= 0 \\
1026(-x^3 + 3x^2 + x - 3) &= 0 \quad | :1026 \\
-x^3 + 3x^2 + x &= 3 \\
-x^3 + 3x^2 + x - 3 &= 0
\end{aligned}$$

Mit Taschenrechner

$$L = \{-1; 1; 3\}$$

44)

$$\frac{5x^2 + 32x + 3}{x^2 + 4x + 3} - 2 = \frac{3x + 9}{x - 1} \quad | \cdot (x^2 + 4x + 3)(x - 1)$$

$$D = R \setminus \{1; -1; -3\}$$

$$(5x^2 + 32x + 3)(x - 1) - 2(x^2 + 4x + 3)(x - 1) = (3x + 9)(x^2 + 4x + 3)$$

$$5x^3 + 32x^2 + 3x - 5x^2 - 32x - 3 - 2(x^3 + 4x^2 + 3x - x^2 - 4x - 3) = 3x^3 + 12x^2 + 9x + 9x^2 + 36x + 27$$

$$5x^3 + 32x^2 + 3x - 5x^2 - 32x - 3 - 2x^3 - 8x^2 - 6x + 2x^2 + 8x + 6 = 3x^3 + 12x^2 + 9x + 9x^2 + 36x + 27$$

$$3x^3 + 21x^2 - 27x + 3 = 3x^3 + 21x^2 + 45x + 27 \quad | -3x^3 \quad | -21x^2$$

$$-27x + 3 = 45x + 27 \quad | -45x \quad | -3$$

$$-72x = 24 \quad | :(-72)$$

$$x = -\frac{1}{3}$$

$$L = \left\{ -\frac{1}{3} \right\}$$

45)

$$\frac{2}{x-1} + \frac{3}{x-2} = \frac{5}{x-3} \quad | \cdot (x-1)(x-2)(x-3)$$

$$D = R \setminus \{1; 2; 3\}$$

$$2(x-2)(x-3) + 3(x-1)(x-3) = 5(x-1)(x-2)$$

$$2(x^2 - 3x - 2x + 6) + 3(x^2 - 3x - x + 3) = 5(x^2 - 2x - x + 2)$$

$$2(x^2 - 5x + 6) + 3(x^2 - 4x + 3) = 5(x^2 - 3x + 2)$$

$$2x^2 - 10x + 12 + 3x^2 - 12x + 9 = 5x^2 - 15x + 10$$

$$5x^2 - 22x + 21 = 5x^2 - 15x + 10 \quad | -5x^2 + 15x - 21$$

$$-7x = -11 \quad | :(-7)$$

$$x = \frac{11}{7}$$

$$L = \left\{ \frac{11}{7} \right\}$$

HN bestimmen:

x-1	x-1		
x-2		x-2	
x-3	x-1		x-3
HN	x-1	x-2	x-3

46)

$$\frac{4}{x+2} - \frac{1}{x-1} = \frac{3}{x+1} \quad | \cdot (x-1)(x+1)(x+2)$$

$$D = \mathbb{R} \setminus \{-1; 1; -2\}$$

$$4(x-1)(x+1) - 1(x+1)(x+2) = 3(x-1)(x+2)$$

$$4(x^2 - 1) - 1(x^2 + 2x + x + 2) = 3(x^2 + 2x - x - 2)$$

$$4x^2 - 4 - x^2 - 2x - x - 2 = 3x^2 + 6x - 3x - 6$$

$$3x^2 - 6 - 3x = 3x^2 + 3x - 6 \quad | -3x^2 \quad | +6 \quad | -3x$$

$$-6x = 0 \quad | :(-6)$$

$$x = 0$$

$$L = \{0\}$$

HN bestimmen:

x+2	x+2		
x-1		x-1	
x+1			x+1
HN	x+2	x-1	x+1

Proben:

Bemerkung zu den Aufgaben 12, 13, 17, 20, 32, 33

Man müsste für alle reelle Zahlen Probe machen und dann müsste IMMER eine falsche Aussage entstehen (links und rechts des Gleichheitszeichens eine unterschiedliche Zahl sein).

Man könnte zur Probe z.B. 3 Zahlen testen

Bemerkung zu den Aufgaben 11, 22, 28

Man müsste für alle reelle Zahlen Probe machen und dann müsste IMMER eine wahre Aussage entstehen (links und rechts des Gleichheitszeichens die gleiche Zahl sein). Man

könnte zur Probe z.B. 3 Zahlen testen.

Lösungen:

1)

$$\frac{12}{3} + \frac{12}{4} = 7$$

$$4 + 3 = 7$$

$$7 = 7 \quad (w)$$

2)

$$\frac{20}{2} - \frac{20}{5} = 6$$

$$10 - 4 = 6$$

$$6 = 6 \quad (w)$$

3)

$$\frac{7 \cdot 2}{10} - \frac{2}{5} = \frac{2}{2}$$

$$\frac{14}{10} - \frac{4}{10} = \frac{2}{2}$$

$$\frac{10}{10} = \frac{2}{2} \quad (w)$$

4)

$$\frac{3}{4} - \frac{1}{2} = \frac{5 \cdot 1}{6} + \frac{7}{12}$$

$$\frac{3}{4} - \frac{1}{8 \cdot 2} = \frac{5}{48} + \frac{7}{12}$$

$$\frac{3}{4} - \frac{1}{16} = \frac{5}{48} + \frac{7}{12} \quad | \cdot 48$$

$$36 - 3 = 5 + 28$$

$$33 = 3 \quad (w)$$

5)

$$\frac{3 \cdot \frac{60}{7}}{5} - \frac{\frac{60}{7}}{6} - \frac{5 \cdot \frac{60}{7}}{12} - \frac{\frac{60}{7}}{15} = \frac{60}{7} - 9$$

$$\frac{180}{7} - \frac{60}{7 \cdot 6} - \frac{300}{12} - \frac{60}{7 \cdot 15} = \frac{60}{7} - 9$$

$$\frac{180}{7 \cdot 5} - \frac{10}{7} - \frac{300}{7 \cdot 12} - \frac{4}{7} = \frac{60}{7} - 9$$

$$\frac{36}{7} - \frac{10}{7} - \frac{50}{14} - \frac{4}{7} = \frac{60}{7} - 9$$

$$\frac{36}{7} - \frac{10}{7} - \frac{25}{7} - \frac{4}{7} = \frac{60}{7} - 9 \quad | \cdot 7$$

$$36 - 10 - 25 - 4 = 60 - 63$$

$$-3 = -3 \quad (w)$$

6)

$$\frac{1}{2} \cdot \frac{11}{4} - \frac{2}{3} \cdot \frac{11}{4} + \frac{3}{4} \cdot \frac{11}{4} - \frac{5}{6} \cdot \frac{11}{4} + \frac{7}{12} \cdot \frac{11}{4} = \frac{11}{12}$$

$$\frac{11}{8} - \frac{11}{6} + \frac{33}{16} - \frac{55}{24} + \frac{77}{48} = \frac{11}{12} \quad | \cdot 48$$

$$66 - 88 + 99 - 110 + 77 = 44$$

$$44 = 44 \quad (w)$$

7)

$$\frac{10+4}{14} + \frac{10-4}{6} = 2$$

$$\frac{14}{14} + \frac{6}{6} = 2$$

$$1+1=2$$

$$2=2 \quad (w)$$

8)

$$\frac{-\frac{13}{2}-3}{18} - \frac{2 \cdot -\frac{13}{2}+7}{8} = \frac{2}{9}$$

$$\frac{-\frac{13}{2}-\frac{6}{2}}{18} - \frac{-13+7}{8} = \frac{2}{9}$$

$$\frac{-19}{18} - \frac{-6}{8} = \frac{2}{9}$$

$$\frac{-19}{2 \cdot 18} + \frac{3}{4} = \frac{2}{9}$$

$$\frac{-19}{36} + \frac{3}{4} = \frac{2}{9} \quad | \cdot 36$$

$$-19+27=8$$

$$8=8 \quad (w)$$

14)

$$\frac{4}{\frac{4}{5}} = 5$$

$$\frac{4 \cdot 5}{4} = 5$$

$$5=5 \quad (w)$$

15)

$$\frac{3}{2 \cdot \frac{3}{8}} = 4$$

$$\frac{3}{\frac{6}{8}} = 4$$

$$\frac{3}{\frac{3}{4}} = 4$$

$$\frac{3 \cdot 4}{3} = 4$$

$$4=4 \quad (w)$$

9)

$$\frac{2+1}{3} - \frac{3 \cdot 2-1}{5} = 2-2$$

$$\frac{3}{3} - \frac{5}{5} = 0$$

$$0=0 \quad (w)$$

10)

$$\frac{3 \cdot 9-5}{4} + \frac{2 \cdot 9-3}{6} - \frac{9+6}{3} = \frac{9-3}{2}$$

$$\frac{22}{4} + \frac{15}{6} - \frac{15}{3} = \frac{6}{2}$$

$$\frac{11}{2} + \frac{5}{2} - 5 = 3$$

$$\frac{16}{2} - 5 = 3$$

$$8-5=3$$

16)

$$\frac{9}{2 \cdot \frac{5}{2}} + 5 = \frac{17}{\frac{5}{2}}$$

$$\frac{9}{5} + 5 = \frac{17 \cdot 2}{5} \quad | \cdot 5$$

$$9+25=35$$

$$34=34 \quad (w)$$

$$18) \quad \frac{1}{\frac{3}{2}} + \frac{1}{3 \cdot \frac{3}{2}} = \frac{8}{9}$$

$$\frac{2}{3} + \frac{1}{\frac{9}{2}} = \frac{8}{9}$$

$$\frac{2}{3} + \frac{2}{9} = \frac{8}{9} \quad | \cdot 9$$

$$6 + 2 = 8 \quad (w)$$

$$19) \quad \frac{3}{4 \cdot \frac{10}{7}} + \frac{3}{6 \cdot \frac{10}{7}} = \frac{7}{8}$$

$$\frac{3}{\frac{40}{7}} + \frac{3}{\frac{60}{7}} = \frac{7}{8}$$

$$\frac{3 \cdot 7}{40} + \frac{3 \cdot 7}{60} = \frac{7}{8}$$

$$\frac{21}{40} + \frac{21}{60} = \frac{7}{8} \quad | \cdot 120$$

$$21 \cdot 3 + 21 \cdot 2 = 7 \cdot 15$$

$$63 + 42 = 105$$

$$105 = 105 \quad (w)$$

$$21) \quad -\frac{1}{3} + \frac{3}{2 \cdot -\frac{5}{9}} + \frac{5}{6} - \frac{4}{3 \cdot -\frac{5}{9}} - \frac{7}{2} = \frac{11}{6 \cdot -\frac{5}{9}}$$

$$-\frac{1}{3} - \frac{3}{\frac{10}{9}} + \frac{5}{6} + \frac{4}{\frac{5}{3}} - \frac{7}{2} = -\frac{11}{\frac{10}{3}}$$

$$-\frac{1}{3} - \frac{3 \cdot 9}{10} + \frac{5}{6} + \frac{4 \cdot 3}{5} - \frac{7}{2} = -\frac{11 \cdot 3}{10}$$

$$-\frac{1}{3} - \frac{27}{10} + \frac{5}{6} + \frac{12}{5} - \frac{7}{2} = -\frac{33}{10} \quad | \cdot 30$$

$$-10 - 81 + 25 + 72 - 105 = -99$$

$$-99 = -99 \quad (w)$$

$$23) \quad \frac{4}{-1+5} = 1$$

$$\frac{4}{4} = 1$$

$$1 = 1 \quad (w)$$

$$24) \quad \frac{7}{\frac{13}{2}-3} = 2$$

$$\frac{7}{\frac{13-6}{2}} = 2$$

$$\frac{7}{\frac{7}{2}} = 2$$

$$\frac{7 \cdot 2}{7} = 2$$

$$2 = 2 \quad (w)$$

$$25) \quad \frac{3}{-\frac{16}{5}+2} + \frac{5}{2} = 0$$

$$\frac{3}{-\frac{16}{5} + \frac{10}{5}} + \frac{5}{2} = 0$$

$$\frac{3}{-\frac{6}{5}} + \frac{5}{2} = 0$$

$$-\frac{3 \cdot 5}{6} + \frac{5}{2} = 0$$

$$-\frac{5}{2} + \frac{5}{2} = 0 \quad (w)$$

$$26) \quad \frac{2}{-5+1} = \frac{3}{-5-1}$$

$$\frac{2}{-4} = -\frac{3}{6}$$

$$-\frac{1}{2} = -\frac{1}{2} \quad (w)$$

$$29) \quad \frac{1}{\frac{13}{4}-3} - 2 = \frac{1}{2 \cdot (\frac{13}{4}-3)}$$

$$\frac{1}{\frac{13-12}{4}} - 2 = \frac{1}{2 \cdot (\frac{13-12}{4})}$$

$$\frac{1}{\frac{1}{4}} - 2 = \frac{1}{2 \cdot \frac{1}{4}}$$

$$\frac{1 \cdot 4}{1} - 2 = \frac{1}{\frac{1}{2}}$$

$$4 - 2 = 2$$

$$2 = 2 \quad (w)$$

30)

$$\frac{15}{2 \cdot 5 + 5} - \frac{15}{4 \cdot 5 + 10} = \frac{19}{9 \cdot 5 - 7}$$

$$\frac{15}{15} - \frac{15}{30} = \frac{19}{38}$$

$$\frac{30}{30} - \frac{15}{30} = \frac{19}{38}$$

$$\frac{15}{30} = \frac{19}{38}$$

$$\frac{1}{2} = \frac{1}{2} \quad (w)$$

34)

$$\frac{(-2) + 6}{-2} = \frac{(-2) + 4}{(-2) + 1}$$

$$\frac{4}{-2} = \frac{2}{-1}$$

$$-2 = -2 \quad (w)$$

37)

$$3 \cdot \left(-\frac{5}{2} \right) + \frac{5}{\left(-\frac{5}{2} \right) + 1} = 3$$

$$\frac{-15}{2} + \frac{5 \cdot 2}{-5} = 3$$

$$\frac{-15}{-5 + 2} + \frac{5 \cdot 2}{-5} = 3$$

$$\frac{-15}{-3} - \frac{5 \cdot 2}{5} = 3$$

$$\frac{15}{-3} - \frac{5 \cdot 2}{5} = 3$$

$$5 - 2 = 3$$

$$3 = 3 \quad (w)$$

31)

$$\frac{5 \cdot (-12)}{2 \cdot (-12) - 2} - \frac{-12}{3 \cdot (-12) - 3} = 2$$

$$\frac{-60}{-26} - \frac{-12}{-39} = 2$$

$$\frac{60}{26} - \frac{12}{39} = 2$$

$$\frac{30}{13} - \frac{4}{13} = 2$$

$$2 = 2 \quad (w)$$

36)

$$\frac{1}{2} + \frac{1}{2} = 1$$

$$\frac{1}{2} + \frac{1}{2} - 1 = 1$$

$$2 - 1 = 1$$

$$1 = 1 \quad (w)$$

38)

$$\frac{3 \cdot 5 + 1}{4 \cdot 5 - 10} + \frac{5 \cdot 5 - 1}{6 \cdot 5 - 15} - \frac{7 \cdot 5 + 5}{10 \cdot 5 - 25} = \frac{8}{5}$$

$$\frac{16}{10} + \frac{24}{15} - \frac{40}{25} = \frac{8}{5}$$

$$\frac{8}{5} + \frac{8}{5} - \frac{8}{5} = \frac{8}{5}$$

$$\frac{8}{5} = \frac{8}{5} \quad (w)$$

39)

$$\frac{3}{5} + \frac{5}{\left(\frac{5}{7}\right)^2} - \frac{3}{\frac{5}{7} - 1} = \frac{2}{\left(\frac{5}{7} - 1\right)^2}$$

$$\frac{3 \cdot 7}{5} + \frac{5}{\frac{25}{49}} - \frac{3}{\frac{5}{7} - \frac{7}{7}} = \frac{2}{\left(\frac{5}{7} - \frac{7}{7}\right)^2}$$

$$\frac{21}{5} + \frac{5 \cdot 49}{25} - \frac{3}{-\frac{2}{7}} = \frac{2}{\left(-\frac{2}{7}\right)^2}$$

$$\frac{21}{5} + \frac{49}{5} + \frac{3 \cdot 7}{2} = \frac{2}{\frac{4}{49}}$$

$$\frac{70}{5} + \frac{21}{2} = \frac{2 \cdot 49}{4}$$

$$14 + \frac{21}{2} = \frac{49}{2}$$

$$\frac{28}{2} + \frac{21}{2} = \frac{49}{2}$$

$$\frac{49}{2} = \frac{49}{2} \quad (w)$$

41)

$$\frac{3}{0+4} - \frac{2}{0-4} = \frac{5 \cdot 0 - 20}{0^2 - 16}$$

$$\frac{3}{4} + \frac{2}{4} = \frac{20}{16}$$

$$\frac{5}{4} = \frac{5}{4} \quad (w)$$

40)

$$\frac{3 + 7 \cdot \frac{10}{9} - 4 - 9 \cdot \frac{10}{9}}{1 + \frac{10}{9}} - \frac{15 - 4 \cdot \left(\frac{10}{9}\right)^2}{1 - \frac{10}{9}} + 6 = \frac{15 - 4 \cdot \left(\frac{10}{9}\right)^2}{1 - \left(\frac{10}{9}\right)^2}$$

$$\frac{3 + \frac{70}{9} - 4 - 10}{\frac{9}{9} + \frac{10}{9}} - \frac{15 - 4 \cdot \frac{100}{81}}{\frac{9}{9} - \frac{10}{9}} + 6 = \frac{15 - 4 \cdot \frac{100}{81}}{1 - \frac{100}{81}}$$

$$\frac{3 + \frac{70}{9} - \frac{-6}{1}}{\frac{19}{9}} - \frac{15 - \frac{400}{81}}{\frac{81}{81} - \frac{100}{81}} + 6 = \frac{15 - \frac{400}{81}}{\frac{81}{81} - \frac{100}{81}}$$

$$\frac{\frac{27}{9} + \frac{70}{9} - 54}{\frac{19}{9}} + 6 = \frac{\frac{15 \cdot 81}{81} - \frac{400}{81}}{\frac{-19}{81}}$$

$$\frac{97}{19} - 48 = \frac{\frac{1215}{81} - \frac{400}{81}}{\frac{-19}{81}}$$

$$\frac{97 \cdot 9}{9 \cdot 19} - 48 = \frac{\frac{1215 - 400}{81}}{\frac{-19}{81}}$$

$$\frac{97}{19} - 48 = -\frac{\frac{815}{81}}{\frac{19}{81}}$$

$$\frac{97 - 48 \cdot 19}{19} = -\frac{815 \cdot 81}{81 \cdot 19}$$

$$\frac{-815}{19} = \frac{-815}{19} \quad (w)$$

42)

$$\frac{3}{2} - \frac{8 \cdot \left(-\frac{1}{4}\right)^2 - 13 \cdot \left(-\frac{1}{4}\right) + 21}{4 \cdot \left(-\frac{1}{4}\right)^2 - 20 \cdot \left(-\frac{1}{4}\right) + 25} + \frac{\left(-\frac{1}{4}\right) + 4}{2 \cdot \left(-\frac{1}{4}\right) - 5} = 0$$

$$\frac{3}{2} - \frac{8 \cdot \frac{1}{16} + 13 \cdot \frac{1}{4} + 21}{4 \cdot \frac{1}{16} + 20 \cdot \frac{1}{4} + 25} + \frac{-\frac{1}{4} + \frac{16}{4}}{-\frac{1}{2} - \frac{10}{2}} = 0$$

$$\frac{3}{2} - \frac{\frac{1}{2} + \frac{13}{4} + 21}{\frac{1}{4} + 5 + 25} + \frac{\frac{15}{4}}{-\frac{11}{2}} = 0$$

$$\frac{3}{2} - \frac{\frac{2}{4} + \frac{13}{4} + 21}{\frac{1}{4} + 30} + \frac{-15 \cdot 2}{4 \cdot 11} = 0$$

$$\frac{3}{2} - \frac{\frac{15}{4} + \frac{84}{4}}{\frac{1}{4} + \frac{120}{4}} + \frac{-15 \cdot 2}{4 \cdot 11} = 0$$

$$\frac{3}{2} - \frac{\frac{99}{4}}{\frac{121}{4}} - \frac{15}{22} = 0$$

$$\frac{3}{2} - \frac{99 \cdot 4}{4 \cdot 121} - \frac{15}{22} = 0$$

$$\frac{3}{2} - \frac{9}{11} - \frac{15}{22} = 0$$

$$\frac{33}{22} - \frac{18}{22} - \frac{15}{22} = 0$$

$$0 = 0 \quad (w)$$

43)

$$\frac{7(3-5)^2}{6 \cdot 3^2 - 6} = \frac{5 \cdot 3 - 1}{3 \cdot 3 + 3} - \frac{3 \cdot 3 - 2}{6 \cdot 3 - 6}$$

$$\frac{7 \cdot 4}{48} = \frac{14}{12} - \frac{7}{12}$$

$$\frac{7}{12} = \frac{7}{6} - \frac{7}{12}$$

$$\frac{7}{12} = \frac{14}{12} - \frac{7}{12}$$

$$\frac{7}{12} = \frac{7}{12} \quad (w)$$

44)

$$\frac{5 \cdot \left(-\frac{1}{3}\right)^2 + 32 \cdot \left(-\frac{1}{3}\right) + 3}{\left(-\frac{1}{3}\right)^2 + 4 \cdot \left(-\frac{1}{3}\right) + 3} - 2 = \frac{3 \cdot \left(-\frac{1}{3}\right) + 9}{\left(-\frac{1}{3}\right) - 1}$$

$$\frac{5 \cdot \frac{1}{9} - \frac{32}{3} + 3}{\frac{1}{9} - \frac{4}{3} + 3} - 2 = \frac{-1+9}{-\frac{1}{3}-1}$$

$$\frac{\frac{5}{9} - \frac{32}{3} + 3}{\frac{1}{9} - \frac{4}{3} + 3} - 2 = \frac{8}{-\frac{4}{3}}$$

$$\frac{\frac{5}{9} - \frac{96}{9} + 3}{\frac{1}{9} - \frac{12}{9} + 3} - 2 = \frac{8 \cdot 3}{-4}$$

$$\frac{-\frac{91}{9} + 3}{-\frac{11}{9} + 3} - 2 = -6$$

$$\frac{-\frac{91}{9} + \frac{27}{9}}{-\frac{11}{9} + \frac{27}{9}} - 2 = -6$$

$$\frac{-\frac{64}{9}}{\frac{16}{9}} - 2 = -6$$

$$-\frac{64 \cdot 9}{9 \cdot 16} - 2 = -6$$

$$-4 - 2 = -6$$

$$-6 = -6 \quad (w)$$

45)

$$\frac{2}{\frac{11}{7}-1} + \frac{3}{\frac{11}{7}-2} = \frac{5}{\frac{11}{7}-3}$$

$$\frac{2}{\frac{11}{7}-\frac{7}{7}} + \frac{3}{\frac{11}{7}-\frac{14}{7}} = \frac{5}{\frac{11}{7}-\frac{21}{7}}$$

$$\frac{2}{\frac{4}{7}} + \frac{3}{-\frac{3}{7}} = \frac{5}{-\frac{10}{7}}$$

$$\frac{7 \cdot 2}{4} - \frac{7 \cdot 3}{3} = -\frac{7 \cdot 5}{10}$$

$$\frac{7}{2} - 7 = -\frac{7}{2}$$

$$\frac{7}{2} - \frac{14}{2} = -\frac{7}{2}$$

$$-\frac{7}{2} = -\frac{7}{2} \quad (w)$$

46)

$$\frac{4}{0+2} - \frac{1}{0-1} = \frac{3}{0+1}$$

$$\frac{4}{2} + \frac{1}{1} = \frac{3}{1}$$

$$\frac{2}{1} + \frac{1}{1} = \frac{3}{1}$$

$$3 = 3 \quad (w)$$

5 ÜBUNGSAUFGABEN MESK 2BKI1

Im Freizeitpark Rust sprint ein Artist aus 25 Metern Höhe in ein Wasserbecken mit der angeblichen Tiefe von 2,70 Meter Tiefe.

- Welche Geschwindigkeit (Kilometer pro Stunde) hat er beim Auftreffen auf die Wasseroberfläche?
- Wie groß ist die (durchschnittliche) Verzögerung (als Mehrfaches der Erdbeschleunigung angeben), die der Artist beim Abbremsen erleidet ?
- Wie lange wirkt die Verzögerung auf den Artisten ein?
- Kann diese Verzögerung von einem Menschen überlebt werden (Recherche im Internet!) oder ist die Angabe von 2,70 Meter Tiefe falsch?

Lösungen:

$$v = a \cdot t \quad (G1)$$

$$s = \frac{1}{2} at^2 \quad (G2)$$

a) Berechnung der Geschwindigkeit ($a = g$)

$$v = g \cdot t \implies t = \frac{v}{ga} \implies t^2 = \frac{v^2}{g^2}$$

In G2) eingesetzt

$$s = \frac{1}{2} gt^2 \implies s = \frac{1}{2} g \cdot \frac{v^2}{g^2} \implies s = \frac{v^2}{2g} \implies v^2 = 2gs \quad (G3) \implies v = \sqrt{2gs} \quad (G4)$$

also konkret:

$$v = \sqrt{2 \cdot 10 \frac{m}{s^2} \cdot 25m} = \sqrt{500 \frac{m^2}{s^2}} = \sqrt{500} \frac{m}{s} = \sqrt{5 \cdot 100} \frac{m}{s} = 10\sqrt{5} \frac{m}{s} = 3,6 \cdot 10\sqrt{5} \frac{km}{h} =$$

$$36\sqrt{5} \frac{km}{h} \approx 80,5 \text{ km/h}$$

b) Berechnung der Verzögerung:

nach (G3) gilt für die Verzögerung a:

$$v^2 = 2as \implies a = \frac{v^2}{2s}$$

also konkret:

$$a = \frac{v^2}{2s} = \frac{\left(\sqrt{500 \frac{m^2}{s^2}}\right)^2}{2 \cdot 2,7m} = \frac{500 \frac{m^2}{s^2}}{5,4m} \approx 92,6 \frac{m}{s^2} = 9,26g$$

c) Dauer der Verzögerung:

nach (G2) gilt für die Zeit t der Verzögerung:

$$t^2 = \frac{2s}{a} \implies t = \sqrt{\frac{2s}{a}}$$

also konkret:

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \cdot 2,7m}{500 \frac{m}{s^2}}} = \sqrt{\frac{5,4m}{500 \frac{m}{s^2}}} = \sqrt{\frac{5,4 \cdot 5,4}{500} s^2} = \sqrt{\frac{29,16}{500}} s \approx 0,24 s$$

d) Recherche im Internet:

Kurzfristig müsste das auszuhalten sein.